AD 257 296

DISCLAIMER NOTICE

DOWN-RANGE ANTI-BALLISTIC MEASUREMENT PROGRAM (DAMP)

ELECTROMAGNETIC WAVE PROPAGATION AND RADIATION CHARACTERISTICS OF ANISOTROPIC PLASMAS

PREPARED FOR
ARMY ROCKET AND GUIDED MISSILE AGENCY
REDSTONE ARSENAL, ALABAMA
UNDER CONTRACT DA-36-034-ORD-3144RD

THIS RESEARCH IS A PART OF PROJEC
DEFENDER, SPONSORED BY THE ADVAN
RESEARCH PROJECTS AGEN
ARPA NO

PREPARED BY
RADIO CORPORATION OF AMERICA
MISSILE AND SURFACE RADAR DIVISION
MOORESTOWN NEW JERSEY

April, 1961

DOWN-RANGE ANTI-BALLISTIC MEASUREMENT PROGRAM (DAMP)

Electromagnetic Wave Propagation and Radiation Characteristics of Anisotropic Plasmas

Prepared for

Army Rocket and Guided Missile Agency
Redstone Arsenal, Alabama

By the

Radio Corporation of America
Missile and Surface Radar Division
Moorestown, New Jersey

Contract DA-36-034-ORD-3144 RD

This research is a part of Project DEFENDER, sponsored by the Advanced Research Projects Agency.

ARPA No. 51

FOREWORD

This report has been prepared to acquaint the reader with some of the results drawn from a study of the electromagnetic properties of plasmas, made by the scientists associated with the RCA Victor, Ltd., Research Laboratories, Montreal, Canada. This work was performed under Contract DA-36-034-ORD-3144RD as part of the Down-Range Anti-Ballistic Measurements Program (DAMP).

ELECTROMAGNETIC WAVE PROPAGATION AND

RADIATION CHARACTERISTICS OF ANISOTROPIC PLASMAS

TABLE OF CONTENTS

ABSTRACT

This report is a study of electromagnetic properties of homogeneous anisotropic plasmas.

The propagation characteristics of electromagnetic waves in anisotropic plasmas are examined for propagation parallel to and perpendicular to the applied d-c magnetic field. The existence of very low frequency pass-bands due to ion effects is noted. Faraday rotation due to the earth's magnetic field is considered for the ionosphere and the plasma sheath of a re-entry vehicle.

A generalized and exact form of Kirchhoff's law is used to obtain the absorptivity of a plasma from its electromagnetic properties. The absorptivity of an isotropic and anisotropic plasma slab is computed for normal incidence and the effects of electron collision frequency, slab thickness, stop-bands and boundary effects on the radiation spectrum is presented.

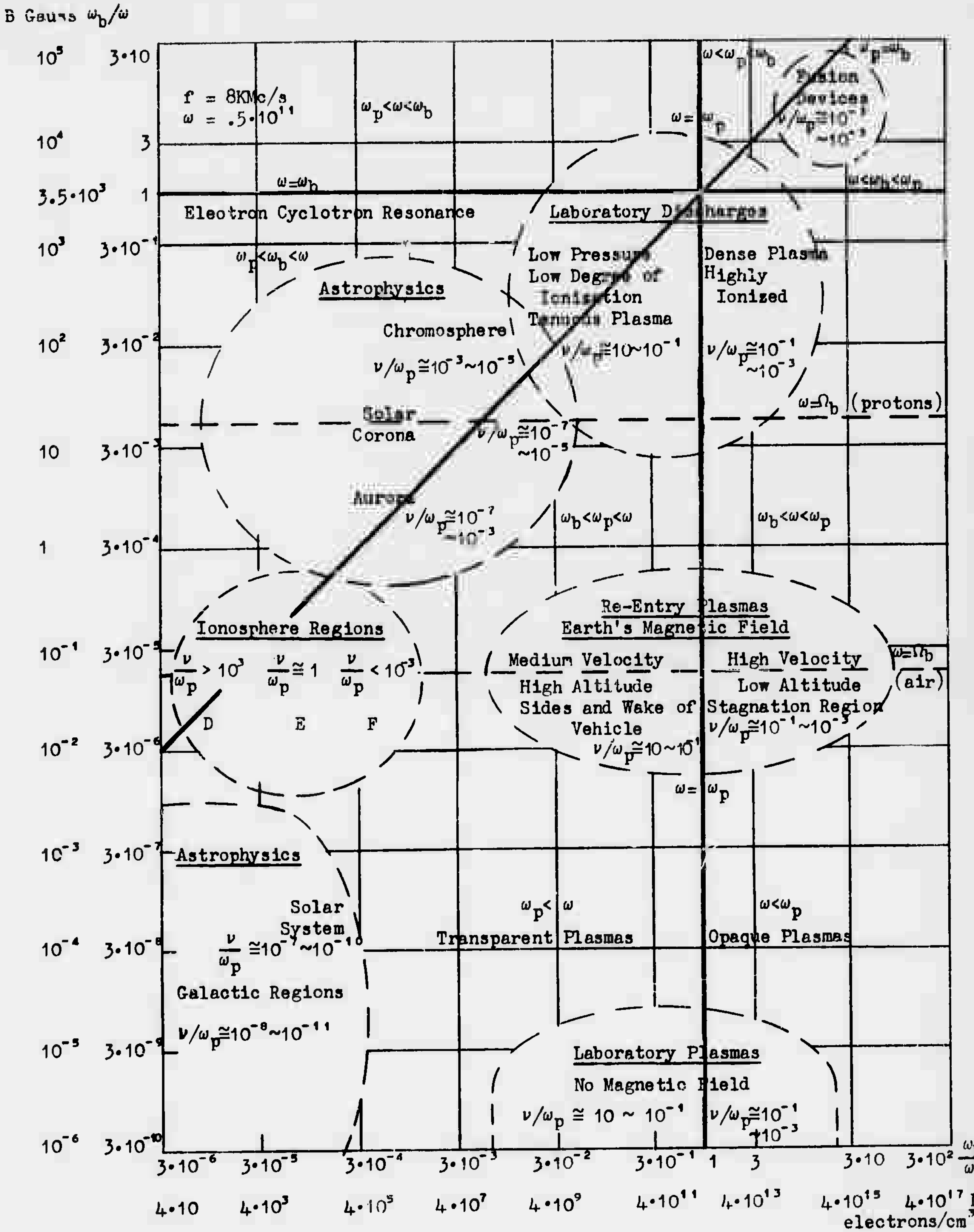

Fig. 1.1 Regions of interest in various types of magnetised plasmas.

ELECTROMAGNETIC WAVE PROPAGATION AND
RADIATION CHARACTERISTICS OF ANISOTROPIC PLASMAS

I. INTRODUCTION

A large percentage of the total matter in the universe consists of plasma, or ionized gases whose net charge is zero. Fig. 1.1 shows the approximate range of electron concentration and magnetic field for some of these plasmas. Plasmas of interest range from fusion devices with very large electron densities and magnetic fields to the most tenuous plasmas encountered in astrophysics. Important parameters when considering electromagnetic propagation and passive radiation from plasmas are the plasma frequency, ω_p, which is directly related to the electron density, and the electron collision frequency, ν. Approximate ratios of ν/ω_p for a number of plasmas of interest are shown in Fig. 1.1. Note that the boundaries of the regions will, in general, extend further than shown, especially towards lower magnetic fields and electron concentrations. Generally, anomalous propagation and enhanced passive radiation is expected in the regions where the electromagnetic frequency, ω, is close to the plasma frequency ω_p or the electron cyclotron frequency, ω_b. Lines of $\omega = \omega_b$ and $\omega = \omega_p$ have been inserted for a frequency of $f = \omega/2\pi = 8$ Kmc/s.

The type of plasma of particular interest in this study is the plasma sheath which forms around a high velocity re-entry vehicle. Propagation of electromagnetic energy in magnetized plasmas is examined and low frequency pass-bands are shown to exist for certain orientations of magnetic field. This opens up the possibility of creating these low frequency pass-bands by means of a fairly high magnetic field carried in the vehicle. A signal would propagate with little reflection or absorption from the sheath at frequencies much lower than the plasma frequency. The Faraday rotation which might take place when propagation is parallel to the earth's magnetic field is examined and shown to be small. The passive microwave radiation spectrum emitted from the plasma sheath of a re-entry vehicle is examined. The radiation spectrum may be considerably different if a magnetic field is carried by the vehicle. The particular case where there is no magnetic field is included. Considerations are also given to the radiation spectrum detected by a microwave radiometer in terms of the emitted spectrum.

The electromagnetic properties of a plasma can be deduced from a knowledge of the dielectric coefficient of a plasma. In the case where the plasma is anisotropic due to an applied d-c magnetic field the dielectric coefficient is a tensor quantity and the propagation characteristics are a function of the orientation of magnetic field. The solution of Maxwell's equations in a uniform plasma shows that there are two possible waves for propagation at any arbitrary angle to the d-c magnetic field. The particular cases of propagation parallel to and perpendicular to the magnetic field are considered.

The equilibrium radiation of a body can be determined from its absorptivity to an incident plane wave by Kirchhoff's law. For the particular case of a uniform plasma slab where dimensions perpendicular to the direction of propagation are infinite the absorptivity is deduced from the propagation characteristics of the plasma by matching the electromagnetic fields at the boundaries. A solution may then be obtained for an arbitrary orientation of the magnetic field by an appropriate choice for the values of the dielectric coefficient. In the present work the absorptivity is computed for the case where the d-c magnetic field is orientated either parallel or perpendicular to the slab boundaries. The absorptivity spectrum as a function of electron collision frequency and slab thickness is presented. The effect of the "stop-bands" of a plasma has a very marked effect on the absorptivity and radiation spectrum. In general, one expects enhanced radiation around the edges of and outside the "stop-bands" as long as the electron collision frequency is not too high. Sharp boundaries have a marked effect on the absorptivity spectrum. For example, internal reflections from the walls can give rise to undulations in the spectrum. The effect of various plasma parameters such as electron density, magnetic field, slab thickness and electron collision frequency is discussed.

II. ELECTROMAGNETIC WAVES IN ANISOTROPIC PLASMAS

2.1 Propagation of Electromagnetic Waves in Anisotropic Plasmas

In the presence of a d-c magnetic field a uniform plasma becomes anisotropic. Consequently, the dielectric coefficient of the plasma must be described in terms of a tensor quantity, i.e. considering a uniform d-c magnetic field along the z-direction[1],

$$K = K_r - jK_i = \begin{pmatrix} \varepsilon_{11} & j\varepsilon_{12} & 0 \\ -j\varepsilon_{12} & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix} = I + \frac{1}{j\omega\varepsilon_0}\sigma \tag{2.1}$$

where: K is the dielectric coefficient with real part K_r and imaginary part K_i

ω is the r-f frequency

ε_0 is the permittivity of free space

ε_{ij} are the elements of the dielectric tensor and

$j = \sqrt{-1}$

$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the unit tensor, and

σ is the conductivity tensor.

The basic wave equations to solve in the case of an anisotropic, homogeneous medium are then:

$$\begin{aligned} \nabla \times \nabla \times \vec{E} &= k^2 K\vec{E} \\ \nabla \times \nabla \times \vec{H} &= k^2 K\vec{H} \end{aligned} \tag{2.2}$$

where: $k = 2\pi/\lambda$ is the wave number.

Since the medium is homogeneous, no gradients in the dielectric coefficient exist. By expanding the above equations in component form it is not too difficult to obtain a solution for an e-m wave propagating in such a medium[1,2]. For a plane wave a valid solution is in terms of the electric field of the form:

$$\vec{E} = \vec{E}_0 \, \varepsilon^{-jk(\vec{n}\cdot\vec{r})} \, \varepsilon^{j\omega t} \tag{2.3}$$

where: $\vec{n}$ is a vector in the direction of propagation of magnitude equal to the refractive index of the plasma.

$\vec{r}$ is the position vector.

The magnitude of the refractive index (n) of the plasma is related to the attenuation constant (α) and phase constant (β) of the propagating electromagnetic wave through:

$$jkn = jkK^{\frac{1}{2}} = (\alpha + j\beta) \tag{2.4}$$

The solutions of Eqns. 2.2 show[1,2,3] that, due to the magnetic field (which causes the anisotropy), two solutions for the refractive index are possible for any direction of propagation of the wave in the plasma. The plasma thus behaves as a doubly refracting medium. The characteristics of the electromagretic waves in the plasma are most readily obtained for the special situations of propagation either along the uniform d-c magnetic field or transverse to (across) the magnetic field. The discussions in this report will be limited to these two cases.

(a) Direction of propagation along the d-c magnetic field.

In this situation, the two values for the dielectric constant (using Eqns. 2.1 - 2.3) are

$$K = n^2 = \varepsilon_{11} \pm \varepsilon_{12} \tag{2.5a}$$

The wave can be shown to consist of two components, one right-hand circularly polarized, the other left-hand circularly polarized. The right-hand circularly polarized (+ sign) corresponds to electron motion transverse to the magnetic field lines. The left-hand circularly polarized wave corresponds to ion motion transverse to the magnetic field lines. These two waves will be called the <u>electron cyclotron</u> and <u>ion cyclotron</u> waves. These are the two waves responsible for the well known <u>Faraday effect</u> - the rotation of plane of polarization of a linearly polarized wave as it propagates through a magneto-ionic medium (see Sec. 2.2).

(b) Direction of propagation transverse to the d-c magnetic field.

The two possible waves in this case depend upon the orientation of the field components of the electromagnetic field relative to the d-c magnetic field.

For the electric vector of the e-m wave parallel to the d-c magnetic field the dielectric constant is:

$$K = \varepsilon_{33} \tag{2.5b}$$

so that the e-m wave is unaffected by the d-c magnetic field. This wave shall be termed the <u>ordinary</u> wave since it has the same characteristics as a wave travelling in an isotropic plasma.

For the magnetic vector of the e-m wave parallel to the d-c magnetic field the dielectric constant is given by:

$$K = \frac{(\varepsilon_{11} - \varepsilon_{12})(\varepsilon_{11} + \varepsilon_{12})}{\varepsilon_{11}} \tag{2.5c}$$

This wave is transverse to the magnetic vector of the electromagnetic field but it is not entirely transverse to the direction of propagation. It is elliptically polarized in the plane perpendicular to the H-vector and contains a component of E in the direction of propagation. As a result of these characteristics this wave shall be called the extraordinary wave. In optics these two types of waves of differing phase velocities which propagate transverse to the d-c magnetic field give rise to the Cotton-Mouton effect.

The normalized attenuation and phase constants for a plane wave can be calculated from[1]:

$$\frac{\alpha}{k} = \left[\frac{1}{2}\left(|K| - K_r\right)\right]^{\frac{1}{2}} \tag{2.6a}$$

$$\frac{\beta}{k} = \left[\frac{1}{2}\left(|K| + K_r\right)\right]^{\frac{1}{2}} \tag{2.6b}$$

where:

$$|K| = \left[K_r^2 + K_i^2\right]^{\frac{1}{2}}$$

The general phenomena can be illustrated by considering a lossless plasma (with collision frequency $\nu = 0$). For a lossless plasma the elements of the dielectric tensor become (including ion effects):[1,3]

$$\epsilon_{11} = 1 - \frac{\omega_p^2}{2\omega^2}\left\{\frac{1}{[1 - \omega_b/\omega][1 + \Omega_b/\omega]} + \frac{1}{[1 + \omega_b/\omega][1 - \Omega_b/\omega]}\right\} \tag{2.7a}$$

$$\epsilon_{12} = - \frac{\omega_p^2}{2\omega^2}\left\{\frac{1}{[1 - \omega_b/\omega][1 + \Omega_b/\omega]} - \frac{1}{[1 + \omega_b/\omega][1 - \Omega_b/\omega]}\right\} \tag{2.7b}$$

$$\epsilon_{33} = 1 - \omega_p^2/\omega^2 \tag{2.7c}$$

where: $\Omega_b = \left|\frac{eB_o}{M}\right|$ = ion cyclotron frequency

$\omega_b = \left|\frac{eB_o}{m}\right|$ = electron cyclotron frequency

$\omega_p = \sqrt{\frac{ne^2}{\varepsilon_o}\left(\frac{1}{m}+\frac{1}{M}\right)}$ = plasma frequency

B_o = d-c magnetic field

m = electron mass

M = ion mass

e = electronic charge

n = number density of electrons

Using these values for the elements of the dielectric coefficient, we can calculate frequencies for which the phase constant $\beta = 0$ i.e. the wave is cut off and will not propagate in the plasma and for which the attenuation constant $\alpha = \infty$, i.e. the wave is completely attenuated[1]. For frequencies between these two limits the wave cannot propagate in the plasma and thus a stop-band exists for these frequencies. The stop bands depend upon orientation of the magnetic field relative to the wave vector, the strength of the d-c magnetic field, the collision frequency, electron density and species of ions in the plasma. A summary of the stop-regions is given in Table 2.1. If in Table 2.1, Ω_b is set equal to zero, then the ions are considered as stationary and hence their effects on the electromagnetic wave are neglected.

In particular note the effects of the ions, namely:

(1) low frequency "windows" exist for both the ion cyclotron and extraordinary waves.

(2) there will be different band-pass regions corresponding to each species of ions. This is the case if the plasma contains a mixture of different ions (e.g. - high temperature air).

TABLE 2.1

STOP BANDS IN A PLASMA (INCLUDING ION EFFECTS)

Type of Wave	Lower Frequency Limit	Upper Frequency Limit
Electron Cyclotron	ω_b	$\sqrt{\left(\frac{\omega_b+\Omega_b}{2}\right)^2+\omega_p^2}+\frac{\omega_b-\Omega_b}{2}$
Ion Cyclotron	Ω_b	$\sqrt{\left(\frac{\omega_b+\Omega_b}{2}\right)^2+\omega_p^2}-\frac{\omega_b-\Omega_b}{2}$
Ordinary	0	ω_p
Extraordinary (two stop regions occur)	$\sqrt{\frac{\omega_b^2+\Omega_b^2+\omega_p^2}{2}}\left[1+\sqrt{1-\frac{4\omega_b\Omega_b(\omega_p^2+\Omega_b\omega_b)}{(\omega_b^2+\Omega_b^2+\omega_p^2)^2}}\right]^{1/2}$ $\sqrt{\frac{\omega_b^2+\Omega_b^2+\omega_p^2}{2}}\left[1-\sqrt{1-\frac{4\omega_b\Omega_b(\omega_p^2+\Omega_b\omega_b)}{(\omega_b^2+\Omega_b^2+\omega_p^2)^2}}\right]^{1/2}$	$\sqrt{\left(\frac{\omega_b+\Omega_b}{2}\right)^2+\omega_p^2}+\frac{\omega_b-\Omega_b}{2}$ $\sqrt{\left(\frac{\omega_b+\Omega_b}{2}\right)^2+\omega_p^2}-\frac{\omega_b-\Omega_b}{2}$

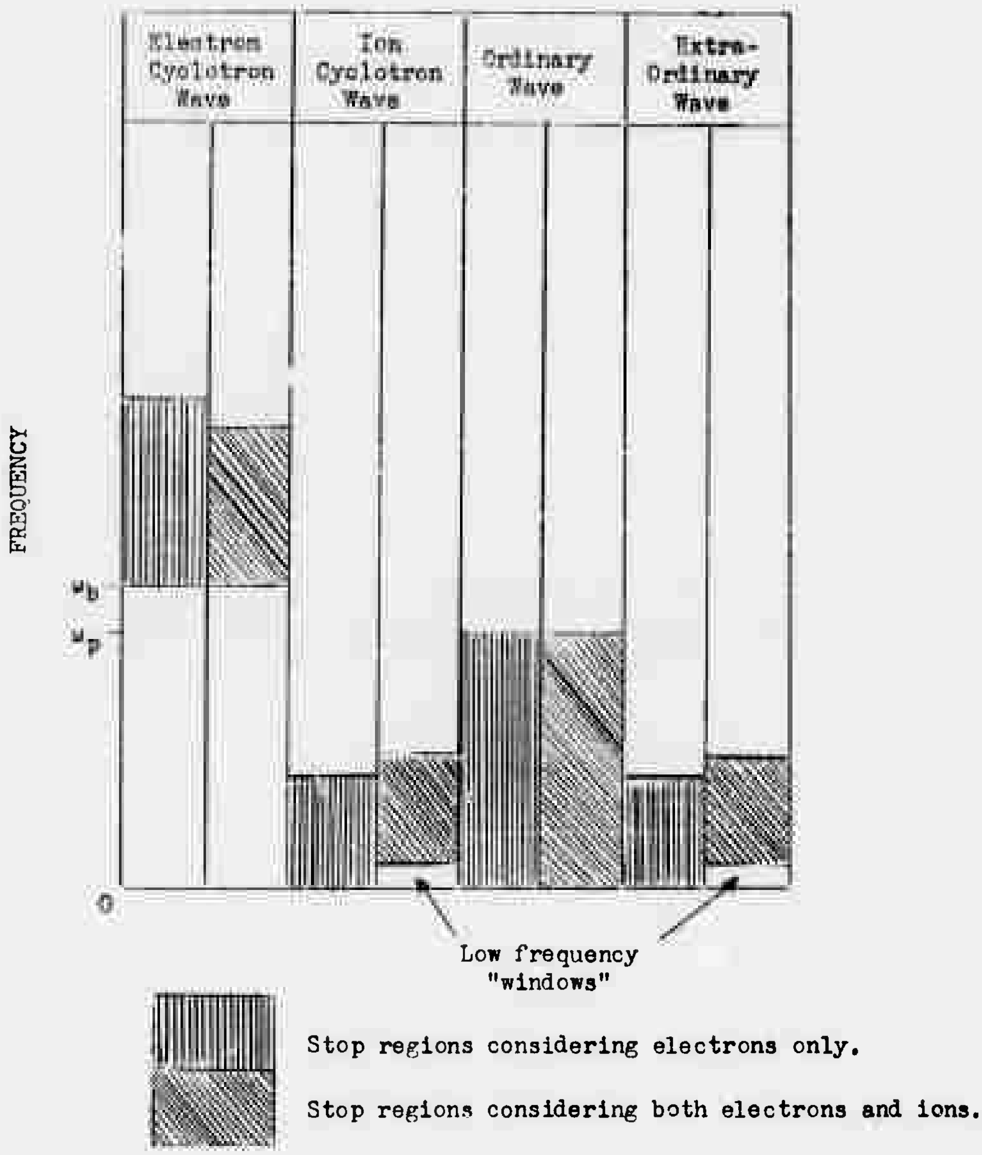

Fig. 2.1 - Stop-band regions for a plasma in the presence of a d-c magnetic field.

A pictorial representation of the stop-bands for the different waves comparing the regions when electron effects only are considered to the regions when ion effects are included is shown in Fig. 2.1.

The effect of losses (i.e. collisions between the plasma constituents) on the propagation of an e-m wave is that the cut-off bands are not as abrupt, but the attenuation increases smoothly. The highly attenuated bands are, however, located in approximately the same regions of frequency as above.

The terms "stop" and "pass" bands are retained in this report for a plasma with collisions, although these terms have little physical meaning when the collision frequency is high ($\nu \sim \omega_p$). The limits of these bands are in all cases taken to be those listed in Table 2.1.

In Table 2.2 are listed the values of the real and imaginary parts of the dielectric coefficient for the various waves. From these expressions the propagation constants for lossy plasma can be calculated using Eq.(2.6).

The normalized attenuation and phase constants (α/k and β/k) for the various waves are shown as functions of the variables[4] $F = \omega/\omega_p$ and $C = \nu/\omega_p$, in Figs. 2.2 to 2.5. The particular case of $\omega_b/\omega_p = 2$ was chosen and ion effects were neglected.

Fig. 2.2 shows the variation of α/k and β/k for the electron cyclotron wave. The effect of the stop-band is very marked. The lower and upper limits of the stop-band (Fig. 2.1) are respectively $\omega = \omega_b$ ($F = 2.0$) and $\omega = \omega_b/2 + \sqrt{(\omega_b/2)^2 + \omega_p^2}$ ($F = 2.414$). Clearly, the plane can be divided into two distinct regions; a "stop" (the lower right side of the plane) and "pass" (the upper left side of the plane) regions whose boundaries are the $F = 2.0$ and $F = 2.414$ contours. These contours are

	Electrons Only			Electrons and Ions		
Wave Type	$K_{r(\nu=0)}$	K_r	K_i	$K_{r(\nu_{ei}=0)}$	K_r	K_i
Cyclotron Waves electron ($K_- = \epsilon_{11} + \epsilon_{12}$)	$1 - \frac{\omega_{pe}^2}{\omega}\frac{1}{\omega-\omega_b}$	$1 - \frac{\omega_{pe}^2}{\omega}\frac{(\omega-\omega_b)}{(\omega-\omega_b)^2+\nu^2}$	$\frac{\omega_{pe}^2}{\omega}\frac{\nu}{(\omega-\omega_b)^2+\nu^2}$	$1 - \frac{\omega_p^2}{\omega^2}\frac{1}{(1-\omega_b/\omega)(1+\Omega_b/\omega)}$	$1 - \left\{\frac{\omega_{pe}^2}{\omega}\frac{(\omega-\omega_b)}{\nu^2+(\omega-\omega_b)^2} + \frac{\omega_{pi}^2}{\omega}\frac{(\omega+\Omega_b)}{\nu_i^2+(\omega+\Omega_b)^2}\right\}$	$\frac{\omega_{pe}^2}{\omega}\frac{\nu}{\nu^2+(\omega-\omega_b)^2} + \frac{\omega_{pi}^2}{\omega}\frac{\nu_i}{\nu_i^2+(\omega+\Omega_b)^2}$
ion ($K_+ = \epsilon_{11} - \epsilon_{12}$)	$1 - \frac{\omega_{pe}^2}{\omega}\frac{1}{\omega+\omega_b}$	$1 - \frac{\omega_{pe}^2}{\omega}\frac{(\omega+\omega_b)}{(\omega+\omega_b)^2+\nu^2}$	$\frac{\omega_{pe}^2}{\omega}\frac{\nu}{(\omega+\omega_b)^2+\nu^2}$	$1 - \frac{\omega_p^2}{\omega^2}\frac{1}{(1+\omega_b/\omega)(1-\Omega_b/\omega)}$	$1 - \left\{\frac{\omega_{pe}^2}{\omega}\frac{(\omega+\omega_b)}{\nu^2+(\omega+\omega_b)^2} + \frac{\omega_{pi}^2}{\omega}\frac{(\omega-\Omega_b)}{\nu_i^2+(\omega-\Omega_b)^2}\right\}$	$\frac{\omega_{pe}^2}{\omega}\frac{\nu}{\nu^2+(\omega+\omega_b)^2} + \frac{\omega_{pi}^2}{\omega}\frac{\nu_i}{\nu_i^2+(\omega-\Omega_b)^2}$
Ordinary Wave ($K_o = \epsilon_{33}$)	$1 - \frac{\omega_{pe}^2}{\omega^2}$	$1 - \frac{\omega_{pe}^2}{\omega^2}\frac{1}{1+\nu^2/\omega^2}$	$\frac{\omega_{pe}^2}{\omega^2}\frac{\nu/\omega}{1+\nu^2/\omega^2}$	$1 - \left(\frac{\omega_p}{\omega}\right)^2$	$1 - \left(\frac{\omega_{pe}^2}{\nu^2+\omega^2} + \frac{\omega_{pi}^2}{\nu_i^2+\omega^2}\right)$	$\frac{\omega_{pe}^2}{\omega}\frac{\nu}{\nu^2+\omega^2} - \frac{\omega_{pi}^2}{\omega}\frac{\nu_i}{\nu_i^2-\omega^2}$
Extraordinary Wave $K_x = \frac{(\epsilon_{11}-\epsilon_{12})(\epsilon_{11}+\epsilon_{12})}{\epsilon_{11}}$ $= \frac{K_+K_-}{(K_+ + K_-)/2}$	$1 - \frac{\omega_{pe}^2}{\omega^2}\frac{(\omega^2-\omega_{pe}^2)}{(\omega^2-\omega_b^2-\omega_{pe}^2)}$	$1 - \frac{\omega_{pe}^2}{\omega^2} \times \frac{(\omega^2-\omega_{pe}^2)(\omega^2-\omega_b^2-\omega_{pe}^2)+\nu^2\omega^2}{[\nu^2-(\omega^2-\omega_b^2-\omega_{pe}^2)]^2+\nu^2[2\omega-\omega_{pe}^2/\omega]^2}$	$\frac{\omega_{pe}^2}{\omega^2}\cdot\nu\omega \times \frac{[\nu^2+\omega^2+\omega_b^2-\omega_{pe}^2]+\frac{\omega_{pe}^2}{\omega^2}(\omega_{pe}^2-\omega^2)}{[\nu^2-(\omega^2-\omega_b^2-\omega_{pe}^2)]^2+\nu^2[2\omega-\omega_{pe}^2/\omega]^2}$	$1 - \left(\frac{\omega_p}{\omega}\right)^2 \times \frac{\omega^2-\omega_b\Omega_b-\omega_p^2}{(\omega^2-\omega_p^2-\omega_b^2-\Omega_b^2)+\frac{\omega_b\Omega_b}{\omega^2}(\omega_p^2+\omega_b\Omega_b)}$	$2\cdot\frac{K_{r-}\lvert K_+^2\rvert + K_{r+}\lvert K_-^2\rvert}{(K_{r-}+K_{r+})^2+(K_{i-}+K_{i+})^2}$	$2\cdot\frac{K_{i+}\lvert K_-^2\rvert + K_{i-}\lvert K_+^2\rvert}{(K_{r-}+K_{r+})^2+(K_{i-}+K_{i+})^2}$
					$\omega_{pi}^2 = \frac{ne^2}{\epsilon_o}\cdot\frac{1}{M}$ $\omega_p^2 = \frac{ne^2}{\epsilon_o}\left(\frac{1}{m}+\frac{1}{M}\right)$	ν = electron collision freq. ν_i = ion collision freq. $\omega_{pe}^2 = \frac{ne^2}{\epsilon_o}\cdot\frac{1}{m}$

Table 2.2. Dielectric coefficient for electromagnetic waves in a uniform anisotropic plasma.

the straight lines $\alpha/k = \beta/k$ when $\nu/\omega_p \ll 1$ but become distorted for larger values of collision frequency. This arises because the edges of the stop-bands correspond exactly to $K_r = 0$ for a collisionless plasma and very closely to $K_r = 0$ for a plasma with a small collision frequency.

Note that in the region of, and below the ion cyclotron frequency, Ω_b (say $F < 10^{-4}$), ion effects, which have been neglected, become important. As $F = \omega/\omega_p$ is increased from 10^{-4} to 10^{-1} (the r-f frequency being well below the plasma frequency) the normalized attenuation constant α/k decreases by a factor of the order of 3; the attenuation constant α, however, increases by a similar factor. Such low frequency electromagnetic waves propagate with very little attenuation inside a plasma. However, since the normalized phase constant, β/k is much larger than unity at these low frequencies the reflections from the plasma boundaries are very high[4]. Such a wave inside a plasma is trapped and constrained to follow the magnetic lines of force (for example the "whistler" mode in the upper atmosphere).

As the r-f frequency of the wave is increased still further beyond $F = 1$ (i.e. around the plasma frequency) the normalized attenuation constant increases and as the lower edge of the stop band is approached both α/k and α increase very rapidly, especially for the low collision frequencies. For example, $C = \nu/\omega_p = 10^{-3}$ gives an increase of a factor of 10^3 in attenuation when $F = \omega/\omega_p$ changes from 1.9 to 2.0. As the lower edge of the stop-band is passed, the value of β/k, and therefore the phase velocity of the wave, decreases very rapidly. Passing through the upper edge of the stop-band into the pass-band (i.e. above the plasma frequency) is characterized by a violent decrease of attenuation constant. Increasing the frequency still further has the effect of making the plasma appear as a slightly lossy dielectric with a normalized phase constant β/k close to

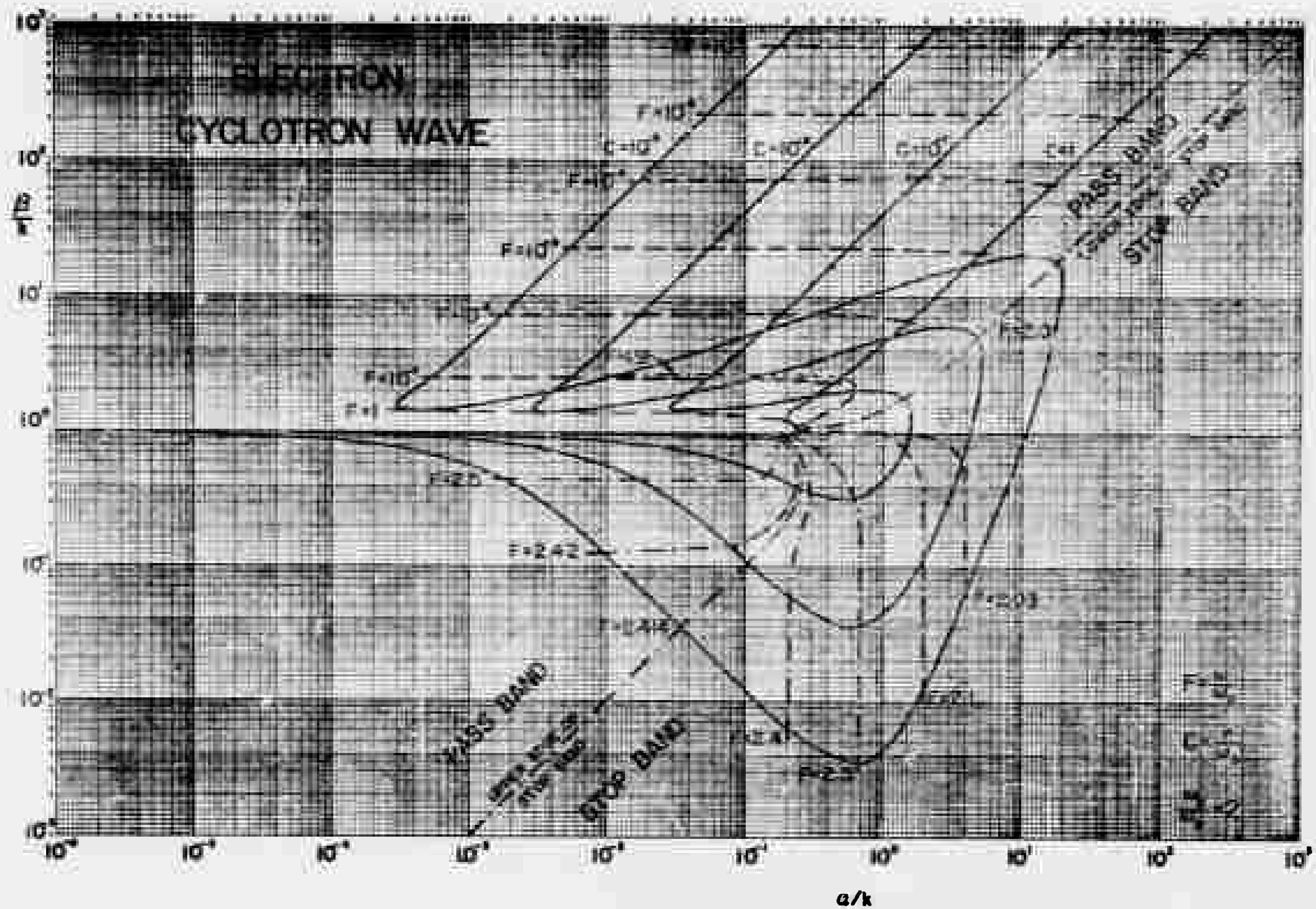

Fig. 2.2 Normalized attenuation and phase constants for the electron cyclotron wave.

unity and with a small attenuation.

Fig. 2.3 shows the propagation constants for the ion cyclotron wave. Again, the plane can be divided into a stop band and a pass band. The boundary between these regions is $F = .414$ (except when $\nu \sim \omega_p$), which corresponds to $\omega = -\omega_b/2 + \sqrt{(\omega_b/2)^2 + \omega_p^2}$. A low frequency wave ($F \ll 1$) propagating in this mode in a plasma (neglecting ion effects) would experience very high attenuation and be quickly damped out. For example, for a collision frequency such that $G = \nu/\omega_p = 10^{-3}$ the ion cyclotron wave would experience an attenuation a factor of 4.10^3 greater than that for the electron cyclotron wave. In the region of the edge of the stop band there is a very rapid decrease in attenuation. The plasma becomes transparent to this wave at a comparatively low frequency, lower than the plasma frequency ($F = 1$).

Fig. 2.4 shows the propagation constants for the ordinary wave in the plasma. It is somewhat similar in shape to that of the ion cyclotron wave, and since it also possesses one low frequency stop-band, most of the previous remarks apply in this case. For low r-f frequencies all the $F = \omega/\omega_p$ contours tend to the line $\alpha/k = \beta/k$. Consequently, in this region the plasma acts like a metallic conductor ($K_i \gg K_r$). The upper edge of the stop band is at the plasma frequency ($F = 1.0$) since this wave is unaffected by the externally applied magnetic field.

The presence of two stop and two pass bands for the extraordinary wave makes the variation of propagation constants rather complicated. Consequently, the diagram is separated into two. Fig. 2.5 shows values of $F = 0$ to 1.0 (which includes the lower stop region) and Fig. 2.6 shows values of $F = 1.0$ to ∞ which includes the upper stop region. This division has the further advantage that this extraordinary wave can be compared to the other wave types rather easily. At the lowest frequencies (again neglecting ion

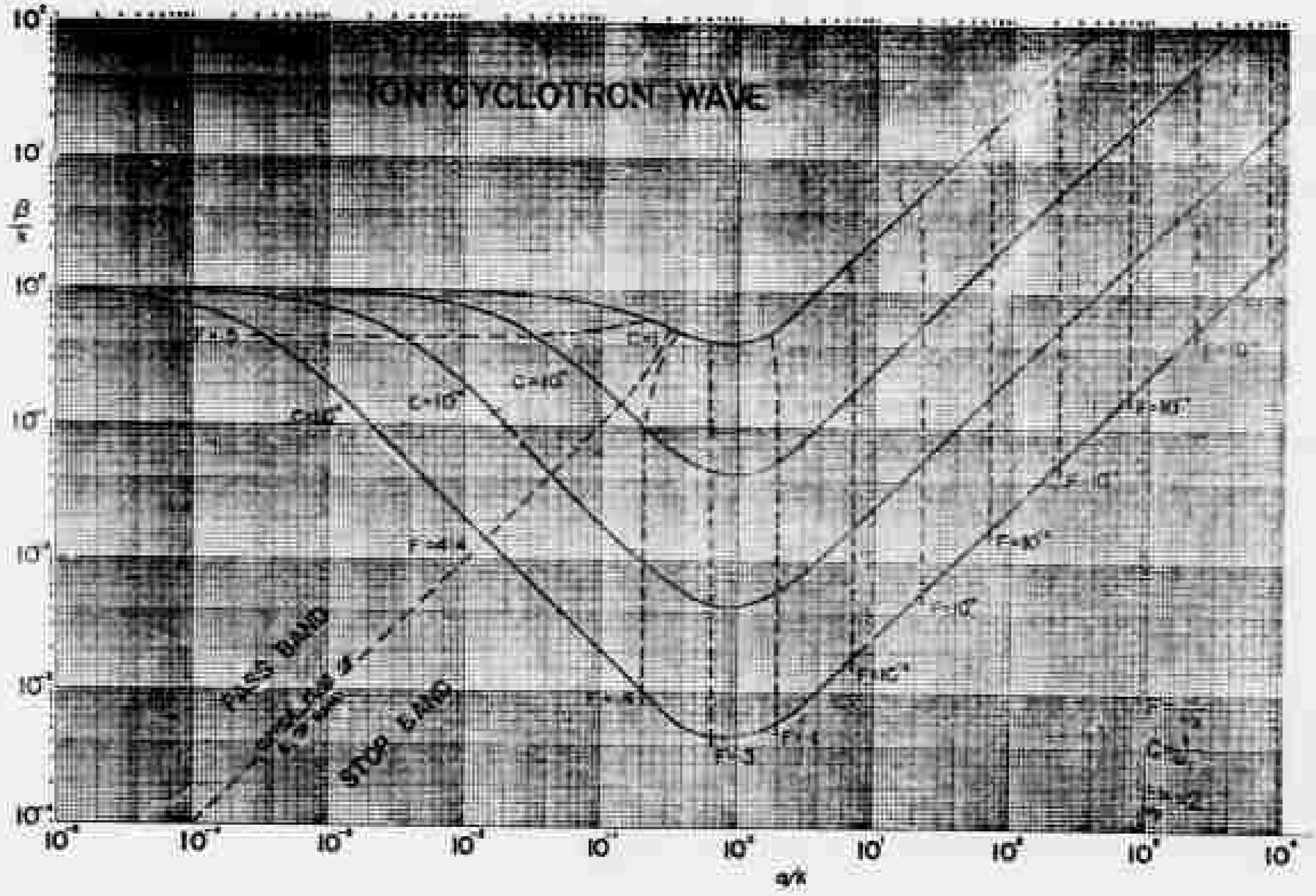

Fig. 2.3 Normalized attenuation and phase constants for the ion cyclotron wave.

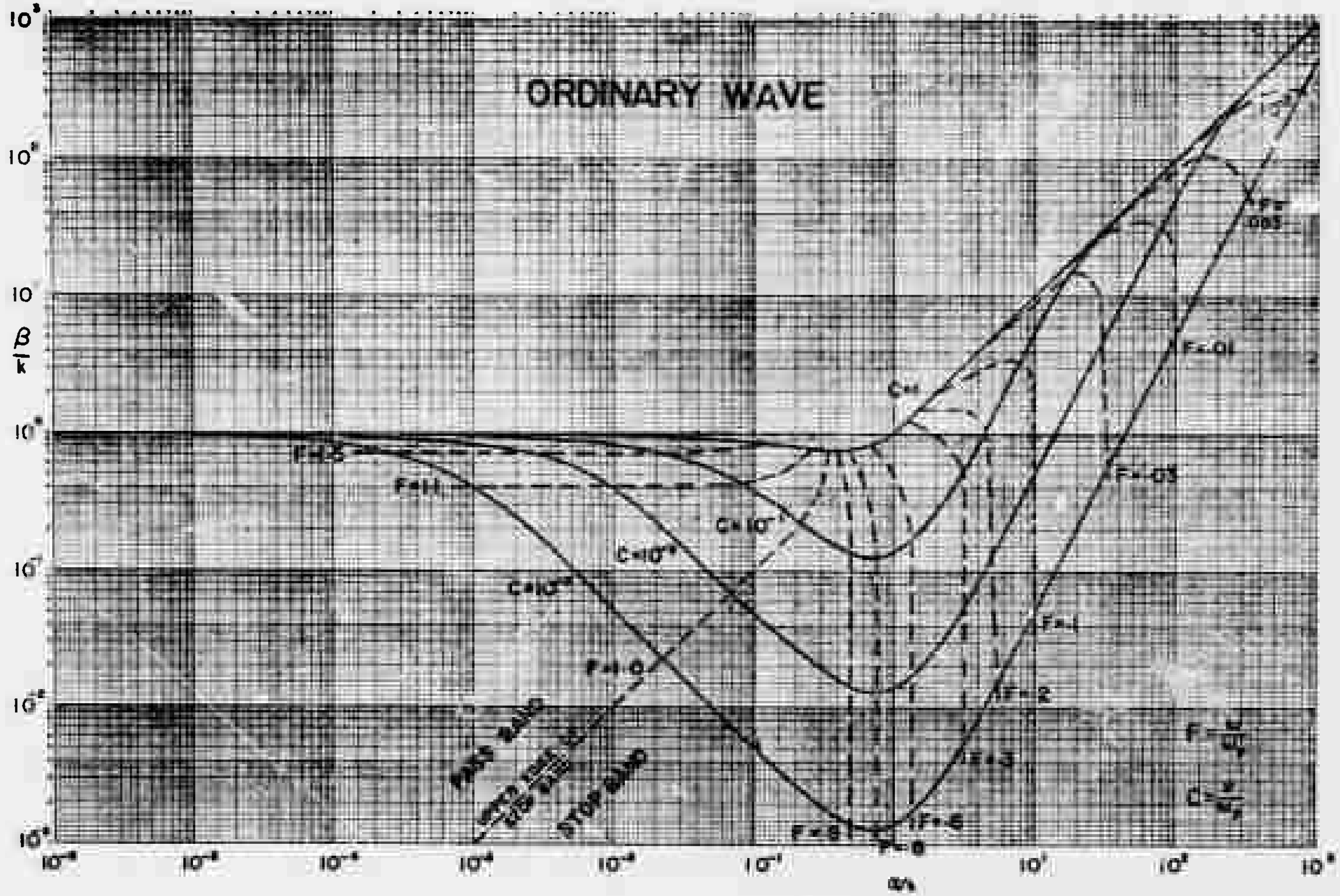

Fig. 2.4 Normalized attenuation and phase constants for the ordinary wave.

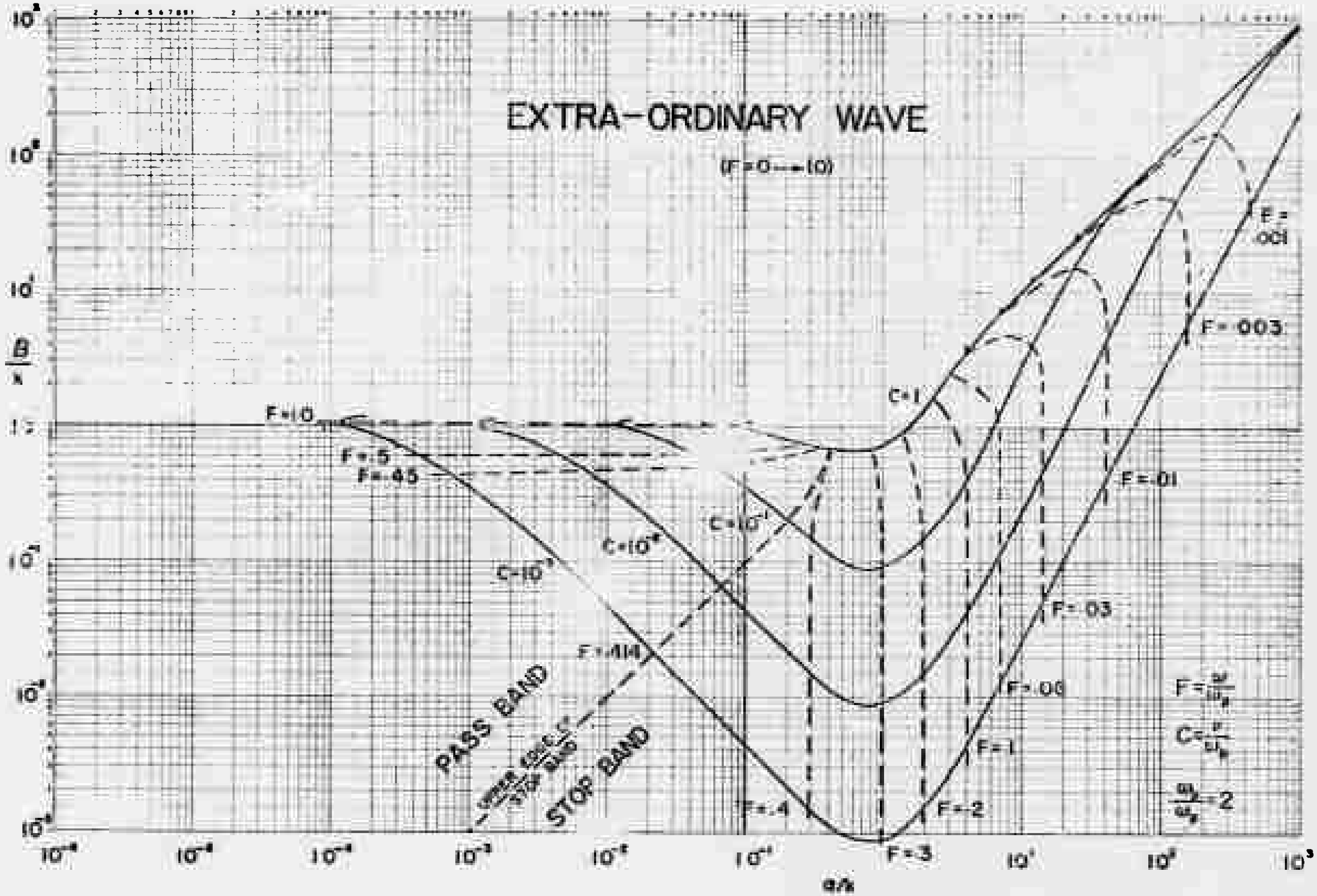

Fig. 2.5 Normalized attenuation and phase constants for the extra-ordinary wave ($F = \omega/\omega_p$ from 0 to 1.0).

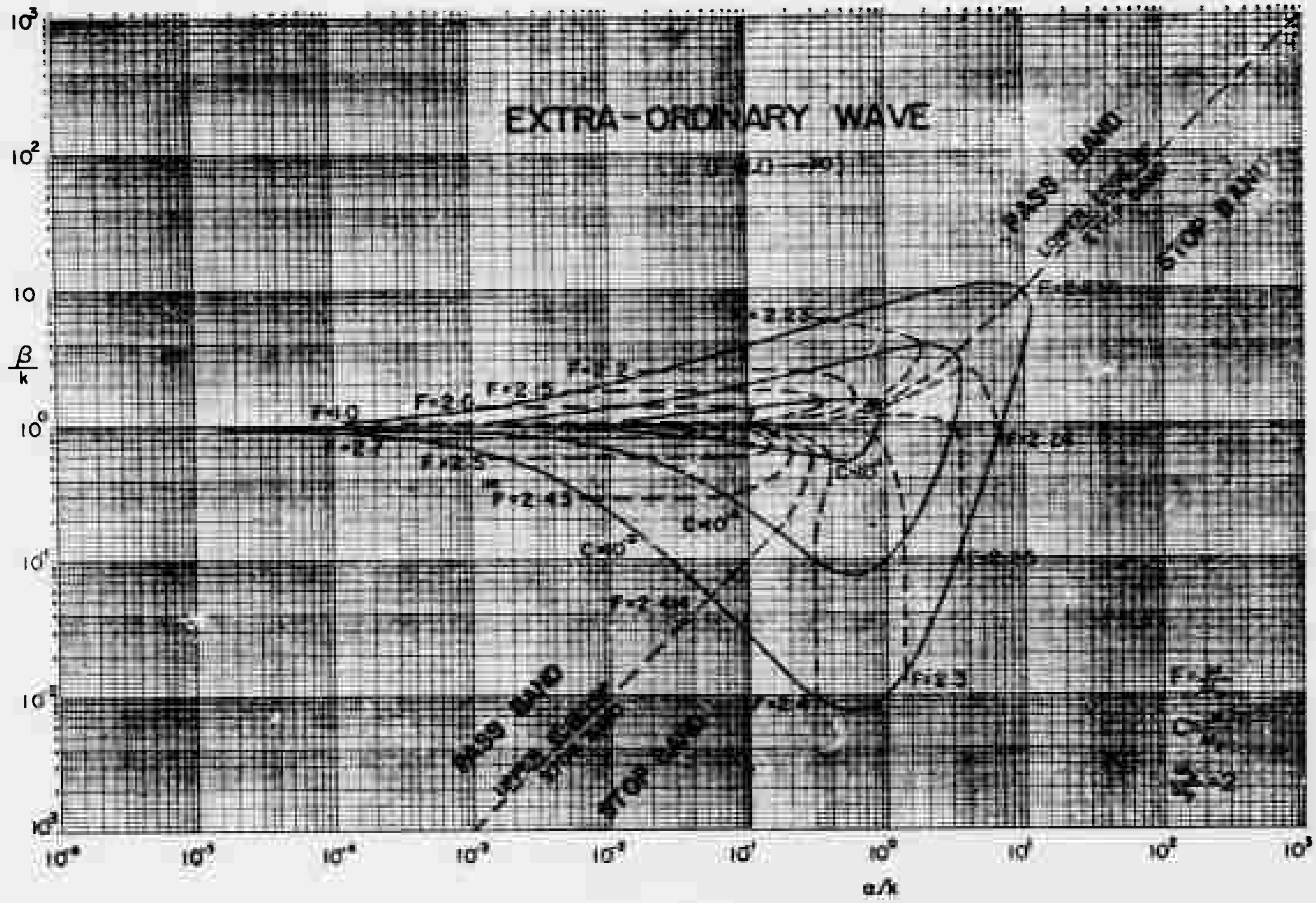

Fig. 2.6 Normalized attenuation and phase constants for the extra-ordinary wave ($F = \omega/\omega_p$ from 1.0 to ∞).

effects) this wave is similar to the ordinary wave. Near the upper edge of the lower stop band it behaves rather like the ion cyclotron wave. For both these waves this boundary occurs at $\omega = -\omega_b/2 + \sqrt{(\omega_b/2)^2 + \omega_p^2}$ (F = .414) and their attenuation and phase constants are similar in this region. Above F = 1 the extraordinary wave behaves much like the electron cyclotron wave. The only major difference is that the lower edge of the upper stop band is now at $\omega = \sqrt{\omega_p + \omega_b^2}$ (F = 2.236) rather than at $\omega = \omega_b$ (F = 2.0).

2.2 Faraday Rotation

Faraday rotation or the rotation of the plane of polarization of a linearly polarized wave propagating in a plasma in a direction parallel to an applied d-c magnetic field, mentioned in the previous section, will in the sequel be derived and applied to the case of e-m waves in ionospheric and re-entry plasmas. As previously shown, two values of the dielectric coefficient are possible depending whether the wave is right-hand or left-hand circularly polarized. The phase velocities for these two waves, the electron cyclotron and the ion cyclotron wave, are given by:

$$v_p = c/K^{1/2} = c/(\varepsilon_{11} \pm \varepsilon_{12})^{1/2} \tag{2.8}$$

where: c is the velocity of light.

Thus the right-hand wave will have, in the plasma, an attenuation constant (α_-) and a phase constant (β_-), while the left-hand wave will have an attenuation constant (α_+) and phase constant (β_+) where

$$\alpha_{\mp} + j\beta_{\mp} = j\omega/v_p = jk(\varepsilon_{11} \pm \varepsilon_{12})^{1/2} \tag{2.9}$$

As is well known, a linearly polarized wave can be split up into two contra-rotating elliptically polarized waves. Thus a plane wave linearly polarized along the x-direction (and propagating in the z-direction) can be written*:

$$E_x = \frac{E_x - jE_y}{2} + \frac{E_x + jE_y}{2} = E_- + E_+$$

where $E_- = \dfrac{E_x - jE_y}{2}$ corresponding to a right hand elliptically polarized wave since $E_y/E_x = -j$.

$E_+ = \dfrac{E_x + jE_y}{2}$ corresponding to a left hand elliptically polarized wave since $E_y/E_x = j$.

Due to the different phase velocities of the two (right-hand and left-hand) circularly polarized waves, the plane of polarization of the resulting linearly polarized wave is rotated as the wave propagates in the plasma. At any point z in the plasma the right-hand polarized wave (assuming initial circular polarization so that: $E_x = E_y = E$) is:

$$E_- = \frac{E_o}{2}(1 - j)e^{-(\alpha_- + j\beta_-)z} \tag{2.10a}$$

Similarly, the left circularly polarized wave is:

$$E_+ = \frac{E_o}{2}(1 + j)e^{-(\alpha_+ + j\beta_+)z} \tag{2.10b}$$

It is assumed that at the point z = 0 in the plasma, the electric vector of the wave was linearly polarized along the x-axis and of amplitude E_o.

For a lossless plasma ($\alpha_- = \alpha_+ = 0$) or for a plasma where the attenuation constants for both waves are approximately the same ($\alpha_- \cong \alpha_+$), the resulting wave in the plasma is given by:

$$E = \frac{E_o}{2}\left\{\left(1 - j\right)e^{-j\beta_- z} + \left(1 + j\right)e^{-j\beta_+ z}\right\}e^{-\alpha z} \tag{2.11}$$

---

* The convention chosen here is more natural in terms of the wave rotation but is opposite to that adopted in many optics books such as for example: Born and Wolf - "Principles of Optics" Pergamon Press (1959) pp.29.

This can be expanded in terms of trigonometric functions and real and imaginary parts separated.

The tangent of the angle (θ) through which the wave has been rotated is given by

$$\tan\theta = \frac{E \text{ imaginary}}{E \text{ real}} = \tan\left[\left(\frac{\beta_- - \beta_+}{2}\right)z\right]$$

$$\theta = \frac{\beta_- - \beta_+}{2} z = \frac{k}{2}\left[(\varepsilon_{11} + \varepsilon_{12})^{1/2} - (\varepsilon_{11} - \varepsilon_{12})^{1/2}\right]z \qquad (2.12)$$

For a lossless plasma ($\nu = 0$) and neglecting the effects of the ions:

$$\varepsilon_{11} \pm \varepsilon_{12} = 1 - \frac{\omega_p^2}{\omega^2}\frac{1}{(1 \mp \omega_b/\omega)} \qquad (2.13)$$

so that the Faraday rotation angle can be readily computed.

For the case of weak gyrotropy ($\omega_b << \omega$) and for r-f frequencies above the plasma frequency ($\omega > \omega_p$), the Faraday rotation angle is given by:

$$\theta \cong \frac{k}{2}\,\frac{\omega_p^2\omega_b}{\omega^3}\,z \qquad (2.14)$$

The Faraday rotation depends on the d-c magnetic field, electron density in the plasma and the distance of propagation of the wave in the medium. For the case of weak fields and lossless media the Faraday rotation is directly proportional to the d-c magnetic field strength and the plasma electron density.

A linearly polarized wave traversing the ionosphere or the ionized plasma sheath of a hypersonic re-entry vehicle may experience a Faraday rotation due to the earth's magnetic field. These effects have been investigated and are shown in Fig. 2.7. A value of the earth's magnetic field of .4 gauss is assumed. The component of the earth's magnetic field parallel to the direction of propagation will in general be less, and so the curves give larger values than would be encountered in practice, except near the magnetic poles. For the ionosphere, an electron density of 3×10^6 electrons/cm^3 has been assumed. As can be seen the Faraday rotation may be appreciable depending on the r-f frequency and the plasma (ionosphere) properties.

The shock front of a re-entry vehicle has a much higher electron density[4] and therefore higher plasma frequency. The assumed electron density for the plasma sheath computations was 3×10^{10} electrons/cm^3 which might correspond to the electron density along the side of the vehicle or in the wake. Near the stagnation region the electron density will be much higher and may be 10^{16}/cm^3 or more[4]. As a result, the Faraday rotation will decrease. This is due to the fact that for Faraday rotation to occur the r-f frequency must be above the plasma frequency or else one of the waves, the ion cyclotron, will not propagate . Thus, Eqn. 2.14 can be written:

$$\theta \cong \frac{k}{2} z \frac{\omega_p^3}{\omega^3} \cdot \frac{\omega_b}{\omega_p} \tag{2.15}$$

Since: ω_p/ω must be less than unity, θ will decrease as the ratio ω_b/ω_p decreases, or alternately as ω_p is increased. From Fig. 2.7 it can be concluded that Faraday rotation will be negligible for a signal propagated through the shock front of a re-entry vehicle.

Faraday rotation of a radio signal passing through the ionosphere due to the earth's magnetic field has proven a problem in high altitude vehicle telemetry. The difficulty is surmounted by the use of circularly polarized antenna systems. The Faraday rotation principle has been used to

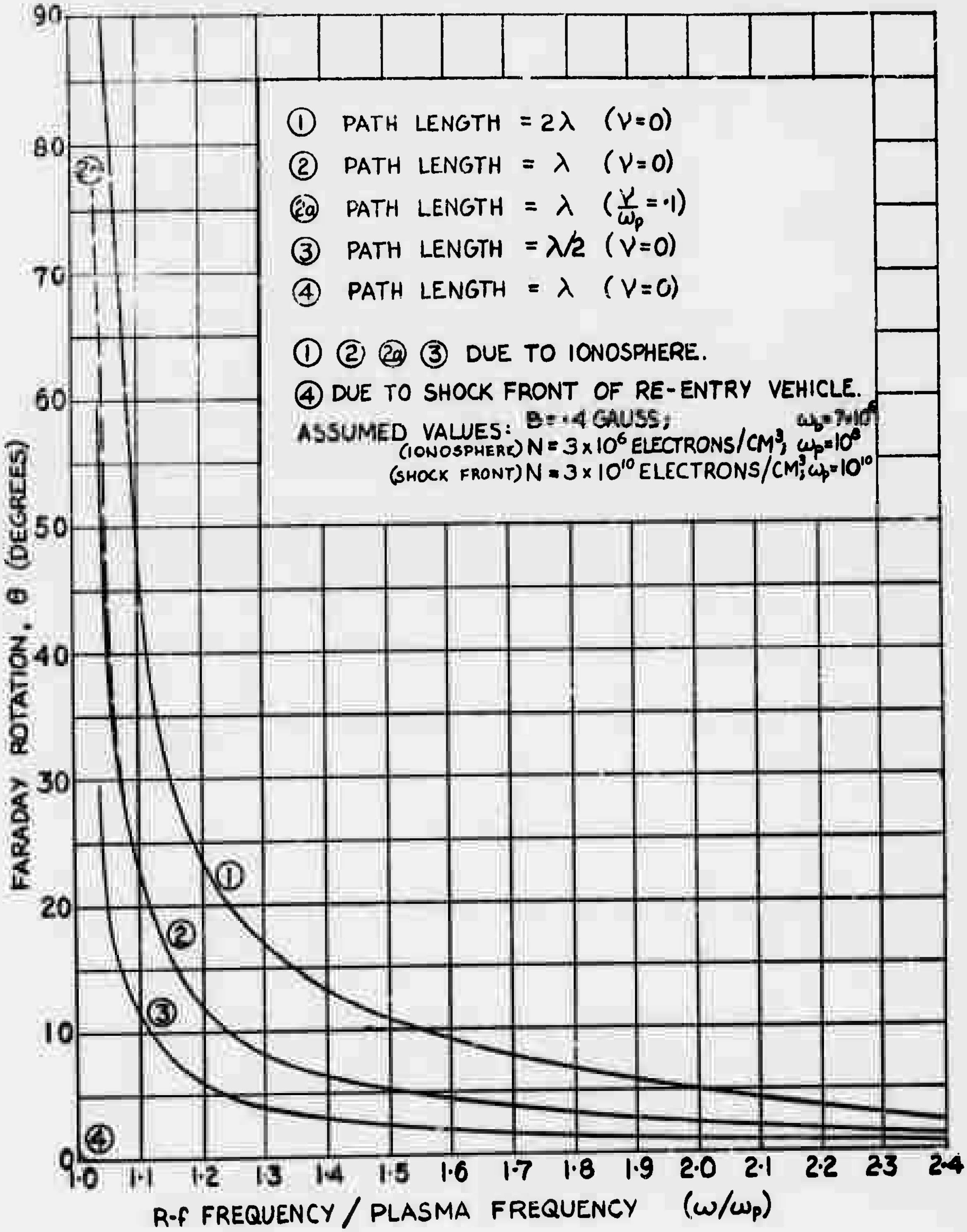

Fig. 2.7 Faraday rotation due to earth's magnetic field.

give a measure of the total electron concentration in the ionosphere[5,23]. One system requires the moon to act as a reflector for a signal from earth. The echo is rotated due to its double passage through the ionosphere. It is possible that this principle may also be used to probe other magnetised plasmas, for example, the sun's corona.

The Faraday rotation has been used to study the decay of discharge plasmas in magnetic fields[6]. The rotation of the plane of polarization can be readily measured and related to the electron density.

Many ferrite microwave devices use the Faraday rotation principle. These devices include isolators, which introduce a high attenuation to waves in one direction only due to the fact that the rotation is in the same sense irrespective of the direction of propagation. Gas discharge isolators, gyrators, attenuators and phase shifting devices are also possible.

2.3 Non-linear Interaction of Electromagnetic Waves in a Plasma

In addition to the effect a plasma has on the properties of a propagating electromagnetic wave, the wave itself, if the electric field strength is sufficiently great, can alter the plasma and hence in turn its own characteristics. For completeness these effects will also be discussed in this report.

Thus, two electromagnetic waves travelling through an ionized region may under certain conditions interact in such a way that a modulation imposed on one of them becomes transferred to the other. This effect is termed the "wave-interaction" or "Luxembourg effect" because the first observations of the effect were made with a set of radio waves interacting in the ionosphere and sent out from the broadcasting station at Luxembourg[7]. The effect has also been observed in the laboratory by Anderson and Goldstein[8].

This effect is qualitatively similar whether the plasma is magneto-active or not and for simplicity, magnetic fields will be neglected in the discussion.

For this interaction to take place the electromagnetic properties of the medium must be a non-linear function of the amplitude of the electromagnetic energy propagating through it. This condition can be satisfied in a plasma since the dielectric properties of the plasma are strong functions of the electron collision frequency. As an e-m wave travels in a plasma it is generally attenuated. The absorption of this energy results in an increase of the collision frequency and a change in the attenuation and phase constants of the plasma. If the amplitude of the incident wave is periodically varied, the temperature of the electrons follows in step and the absorption of the wanted wave also varies periodically. In this manner the modulation of the interacting wave becomes superimposed on the wanted wave.

One should also note that the non-linearity associated with the dependence of the effective collision frequency on the field strength of a strong radio wave will affect the propagation of this wave in the plasma. That is to say, the wave suffers a form of self-distortion or "self-effect" when the wave is not modulated and a form of self modulation if the wave is modulated.

In addition to the effects there is another type of effect, which can occur for a wave propagating in a non-uniform plasma or in any magneto-plasma and is due to the dependence of the electron concentration of the plasma on the field strength associated with the electromagnetic wave. A simple quantitative explanation of these phenomena is now presented.

(a) "Cross-Modulation" and Self Effects

An electromagnetic wave propagating in a plasma, in which the real part of the conductivity is finite, will be attenuated. The attenuation is due to the absorption of power from the wave by the electrons whose kinetic energy is increased. The electrons, in turn, lose some of their energy by collisions with the plasma constituents (neutral particles, ions, electrons). In this way, the plasma tends to attain an equilibrium temperature for all the species. If one considers an average electron, then the change of electron energy, u, with time will depend on the difference between the energy absorbed by each electron per unit time from the electromagnetic wave, P_e, and the energy lost by the electrons due to collisions with the other gas species. Thus the energy equation becomes[1]

$$\frac{du}{dt} + \xi\nu(u - \bar{u}_g) = P_e \tag{2.16}$$

where: ν is the frequency of electron collisions with gas molecules and ions.

ξ is the fractional energy loss per collision for electron-atom and electron-ion collisions.

$\bar{u}_g$ is the average energy of the gas molecules and ions (assumed equal).

Considering the right-hand side of the equation the power per unit area carried by a plane wave as it travels in the z-direction is, according to Eqn.(2.3) and the Poynting theorem:

$$P = P_o e^{-2\alpha z} = \frac{1}{2}\,\mathrm{Re}(E \times H^*) = \frac{1}{2}\,\frac{|E|^2}{Z} \tag{2.17}$$

where: P_o is the incident power per unit area and Z is the characteristic impedance of the plasma, very nearly equal to Z_o, the characteristic free space impedance. Hence the power absorbed in a unit cross-section of thickness dz is:

$$dP = -2\alpha P\, dz \tag{2.18}$$

This element of volume of plasma contains ndz electrons, so that the average energy, $\wp_e$, per second absorbed by each electron is:

$$\wp_e = -\frac{dP}{ndz} = \frac{2\alpha P}{n} \qquad (2.19)$$

For a slightly ionized plasma ($\omega^2 >> \omega_p^2; \omega^2 >> \nu^2$) the attenuation constant α is:

$$\alpha = \left(\frac{\omega_p}{\omega}\right)^2 \frac{\nu}{2c} \qquad (2.20)$$

Hence the average energy absorbed by each electron per second can be written according to Eqn.(2.18)(2.20) and (2.19)

$$\wp_e = \frac{e^2\nu}{m\omega^2} \cdot \frac{1}{2} |E|^2 \qquad (2.21)$$

Note that if the electric field of the wave (E) is a function of time, then the energy absorbed by each electron ($\wp_e$) is also a function of time.

In order to determine the variation of collision frequency with time, it is necessary to relate the electron energy, u, to the collision frequency. Assuming that the electron energy is proportional to the square of the average collision frequency. Then

$$u = A\nu^2 \qquad (2.22)$$

where: A is a constant when a constant mean free path is assumed.

Using Eqns. (2.21) and (2.22) Eqn. (2.16) becomes in terms of the collision frequency

$$\frac{d\nu}{dt} + \frac{\xi}{2}(\nu^2 - \nu_o^2) = \frac{e^2}{4m\omega^2 A} |E_o|^2 \qquad (2.23)$$

where: $\bar{u}_g = A\nu_o^2$ and ν_o is the collision frequency in the absence of the applied external wave.

Consider now a modulated wave travelling in the plasma with its electric field described by

$$E = E_0(1 + M\cos pt) \tag{2.24}$$

where: M is the percentage modulation

p is the modulation frequency

Equation (2.23) may be solved if one assumes that the variations of the collision frequency due to the impressed wave are not very great (this is always true in the ionosphere, for example, since the field strengths applied to the ionosphere are small, even with the most powerful radio transmitters). Then the factor $(\nu^2 - \nu_0^2)$ can be written as

$$\nu^2 - \nu_0^2 = (\nu + \nu_0)(\nu - \nu_0) \cong 2\nu_0(\nu - \nu_0) \tag{2.25}$$

and Eqn. (2.23) becomes

$$\frac{d(\nu - \nu_0)}{dt} + \nu_0\xi(\nu - \nu_0) = \frac{e^2E_0^2}{2m\omega^2 A}(1 + M\cos pt) \tag{2.26}$$

The steady state solution of this equation is

$$\nu - \nu_0 = \frac{e^2E_0^2}{2m\omega^2 A} \cdot \frac{1}{\nu_0\xi}\left[1 + \frac{M}{(1 + (p/\nu_0\xi))^{1/2}}\cos(pt - \phi)\right] \tag{2.27}$$

where: $\phi = \tan^{-1}(p/\xi\nu)$

Eqn. (2.27) determines the change of collision frequency in time due to the modulated wave and hence the change in attenuation of any other wave passing through the same region of plasma.

(b) Effect of Field on Electron Concentration

For a wave propagating in a non-uniform plasma or in a uniform anisotropic plasma, the electric field associated with the wave is, in general, not divergenceless [9,1] i.e. $\nabla\vec{E} \neq 0$. This field is then not purely transverse to the direction of propagation but has a component of the electric vector in the direction of propagation. Such an electric field will produce an electron charge gradient parallel to the wave front which varies periodically at the periodicity of the travelling wave. It will thus give rise to non-uniformities in the plasma. For a wave in such a medium

$$\nabla \cdot \vec{E} = \rho/\varepsilon_o \tag{2.28}$$

where: ρ is the charge density,

and in a non-uniform medium or a uniform anisotropic medium,

$$\rho = e\Delta N \tag{2.29}$$

where: ΔN is the change in electron concentration.

Thus the effect of the non-uniformity or anisotropy of the plasma can be considered as a change in electron concentration, which depends on the electric field since $\nabla \cdot \vec{D} = \nabla \cdot \varepsilon_o \overleftrightarrow{K}\vec{E} = 0$ then

$$\Delta N = \frac{\varepsilon_o}{e}\, \nabla \cdot \vec{E} = -\frac{\varepsilon_o}{e} \cdot \frac{\vec{E} \cdot \nabla\overleftrightarrow{K}}{\overleftrightarrow{K}} \tag{2.30}$$

The change in electron concentration due to the wave results in a change in the dielectric coefficient by an equivalent amount[9,1] (considering collisions).

$$\Delta K = \frac{1}{j\omega\varepsilon_o} \cdot \frac{e^2}{m} \cdot \frac{\Delta N}{\nu + j\omega} = \frac{e}{m\omega} \cdot \frac{\vec{E} \cdot \nabla\overleftrightarrow{K}}{\overleftrightarrow{K}} \cdot \frac{1}{\omega - j\nu} \tag{2.31}$$

For a plane elliptically polarized wave of the form

$$\vec{E} = (\vec{E}_a + j\vec{E}_b)e^{j\omega t}\ e^{-jk(\vec{n}\cdot\vec{r})} \tag{2.32}$$

where: $\vec{n}$ is a vector in the direction of propagation of magnitude equal to the refractive index of the plasma.

$\vec{r}$ is the position vector.

$$\nabla\cdot\vec{E} = -jk[\vec{n}\cdot\vec{E}_a + j\vec{n}\cdot\vec{E}_b]e^{j\omega t}\ e^{-jk(\vec{n}\cdot\vec{r})} \tag{2.33}$$

Note that ΔN depends on the components of the field in the direction of propagation $(\vec{n})$.

Only for the extraordinary wave is $\nabla\cdot\vec{E} \neq 0$ and this is the only wave with a component of the electric vector in the direction of propagation. For simplicity, let the direction of propagation of the extraordinary wave be along the z direction (magnetic vector of the extraordinary wave along the x-direction). Then Eqn. (2.33) gives:

$$\nabla\cdot\vec{E} = -jkn_z\ E_z \tag{2.34}$$

The ellipticity of the extraordinary wave is defined as the ratio of the electric field in the direction of propagation to the electric field transverse to the direction of propagation.

The wave equation becomes[1]

$$\vec{i}[(n_y^2 - \varepsilon_{11})E_z - (n_z n_y + j\varepsilon_{12})E_y] + \vec{j}[n_z n_y - j\varepsilon_{12})E_z + (n_z^2 - \varepsilon_{11})E_y] = 0$$

and it follows that

$$\frac{E_z}{E_y} = -j\,\frac{\varepsilon_{12}}{\varepsilon_{11}} \tag{2.35}$$

so that from Eqn. (2.34)

$$\nabla\cdot\vec{E} = -kn_z\,\frac{\varepsilon_{12}}{\varepsilon_{11}}\,E_y \tag{2.36}$$

Hence
$$\frac{\Delta N}{N} = - \frac{\varepsilon_0}{eN} k n_z \frac{\varepsilon_{12}}{\varepsilon_{11}} E_y \tag{2.37}$$

The change in electron concentration thus depends on the dielectric coefficient tensor and is linear with respect to the field. The other three waves considered in section 2.1, the ordinary, and the cyclotron waves do not exhibit this effect since they do not have a component of electric field in the direction of propagation.

III. ELECTROMAGNETIC RADIATION FROM PLASMAS

3.1 Introduction

In this section the radiation spectrum from an anisotropic plasma is derived from its electromagnetic properties. This approach is based on a general formulation of Kirchhoff's law for the energy radiated by a body.

Rytov[10] and Levin[11] have shown that the emissive power of a body at a frequency ω is given by

$$P_\omega = \frac{KT}{2\pi\lambda^2} A_\omega \tag{3.1}$$

where: A_ω is the power absorptivity of the body (an absorptivity of unity corresponds to a black body).

K is Boltzmann's constant,

T is the temperature of the body and

λ the free-space wavelength.

P_ω is the radiated power per unit frequency interval, per unit solid angle, per unit surface area for a given polarization.

This general expression is valid for any degree of absorptivity while the other form of Kirchhoff's law, $\varepsilon_\omega = \alpha_\omega I_{o\omega}$ (where $I_{o\omega}$ is the black body intensity of radiation, ε_ω and α_ω are emissivity and absorption coefficients, respectively, of the body) describes the equilibrium radiation within the body, and is only valid for small values of absorption coefficient, α_ω. The equilibrium radiation spectrum of a body can then be predicted from the spectrum of the absorptivity. The absorptivity, A_ω, is defined as the fraction of energy absorbed by a body when a plane polarized electromagnetic wave is incident upon it, and is given by[10,11,12]

$$A_\omega = \frac{\int_V \frac{1}{2}\mathrm{Re}(\vec{J}\cdot\vec{E}^*)dv}{\int_S \frac{1}{2}\mathrm{Re}(\vec{E}_i\times\vec{H}_i^*)ds} \tag{3.2}$$

where: $\vec{J}$ and $\vec{E}$ are the current density and the electric field in the body due to an incident electromagnetic field ($\vec{E}_i$ and $\vec{H}_i$) on the body. The numerator which represents the energy absorbed by the body and dissipated as joule losses can in principle be deduced from the electromagnetic properties of the body. The denominator is the energy incident on the body. The evaluation of A_ω from Eqn. (3.2) is a boundary-value problem which has only been solved for very simple geometries[9,11]. Such a solution would be even more complicated for anisotropic and inhomogeneous bodies.

Using a simple model, the radiation spectrum for a plasma in equilibrium in a constant d-c magnetic field will be derived. The plasma geometry of a slab, whose dimensions are infinite along the x-y axis (Fig. 3.1) and of thickness d along the z axis, is chosen. This configuration of plasma permits diffraction effects, which are important when the plasma dimensions are not much greater than the wavelength of the radiation, to be neglected. It is assumed that the plasma is in thermal equilibrium and the electron density is uniform throughout the plasma, implying infinitely sharp boundaries.

Propagation in an anisotropic plasma in any direction can, using Maxwell's equations, be analyzed into the propagation of two waves, whose properties are a function of the orientation and magnitude of the magnetic field. For mathematical simplicity orientations of magnetic field, parallel to and perpendicular to the direction of propagation are chosen. The resulting waves are the electron and ion cyclotron waves for propagation along

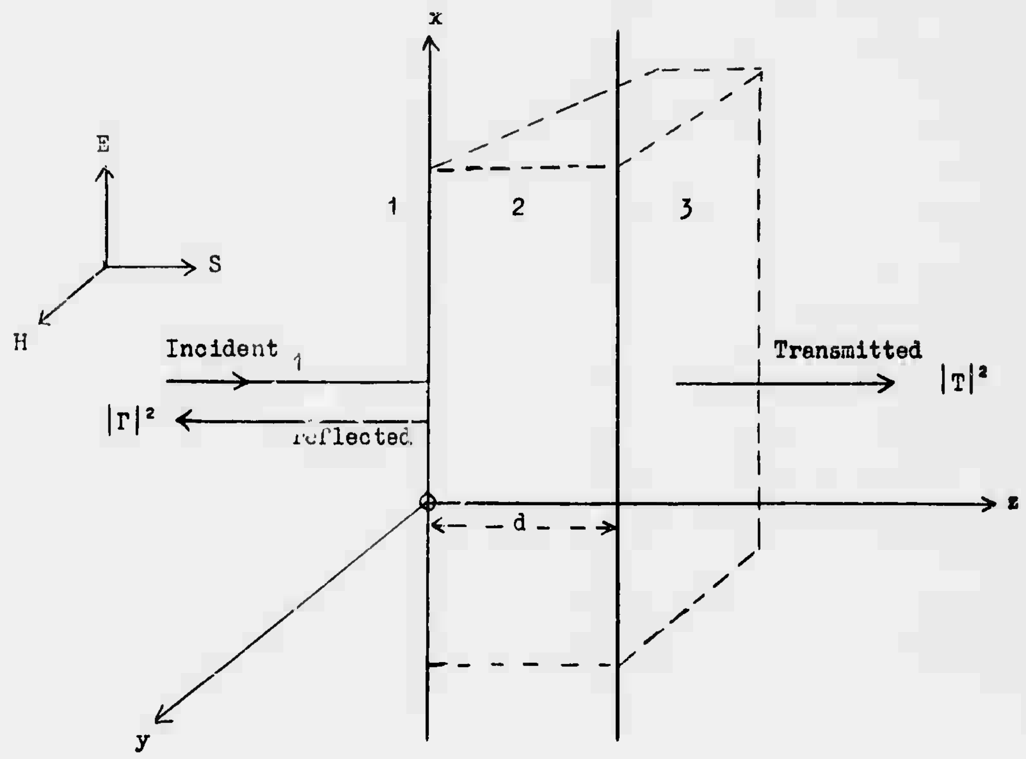

Fig. 3.1a Coordinate system used in calculation of A_ω.

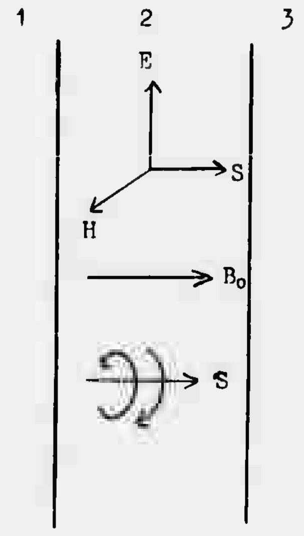

Fig. 3.1b Electron and ion cyclotron waves.

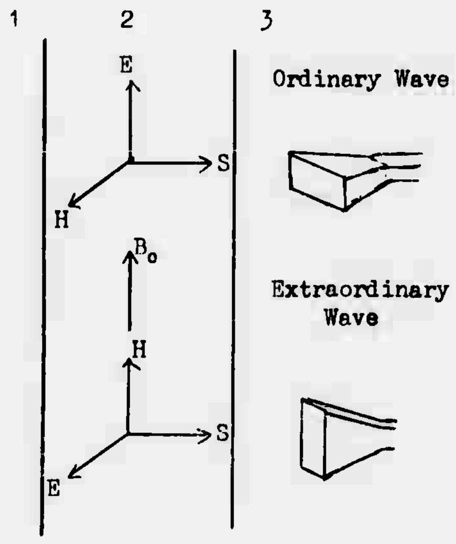

Fig. 3.1c Ordinary and extra-ordinary waves.

the magnetic field and the ordinary and extra-ordinary waves for propagation transverse to the magnetic field (see section 2.1).

The power absorptivity A_ω may be expressed as

$$A_\omega = 1 - |\Gamma|^2 - |T|^2 \tag{3.3}$$

where $|\Gamma|^2$ and $|T|^2$ are the reflectivity and transmissivity respectively of a plane wave incident normally on the slab surface. $|\Gamma|^2$ and $|T|^2$ are evaluated by matching at the boundaries the incident and transmitted fields with fields in the plasma. In this way interval reflections of the e-m waves within the plasma are taken into account. Since only normal incidence is considered nothing can be deduced about the radiation incident at angle other than normal to the surface.

The effect of varying the electron collision frequency and the thickness of the slab are both investigated and shown to change the radiation spectrum radically. The effect of sharp boundaries, which give rise to undulations in the spectrum are discussed. The effects of changing the electron density and the magnetic field are also examined.

3.2 Absorptivity

Let a plane electromagnetic wave be normally incident on the plasma slab with its E-vector along the x-axis (as in Fig. 3.1). On the free-space side of the first plasma boundary, the fields are:

$$\begin{aligned} E_{xi} &= E_0\, e^{j\omega t - \gamma_0 z} & \qquad E_y &= E_z = 0 \\ H_{yi} &= \frac{E_{xi}}{Z_0} = \frac{E_0}{Z_0}\, e^{j\omega t - \gamma_0 z} & \qquad H_x &= H_y = 0 \end{aligned} \tag{3.4}$$

where: $Z_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ is the free-space impedance

ε_0 is the permittivity of free-space.

μ_0 is the permeability of free-space.

γ_0 is the propagation constant in free-space.

$\gamma_0 = jk$ where $k = 2\pi/\lambda$ (λ is the wavelength).

If an amplitude reflection coefficient Γ_0 is defined for the boundary 1-2 (see Fig. 3.1) then the reflected plane wave may be expressed as:

$$E_{xr} = \Gamma_0 E_0 e^{j\omega t+\gamma_0 z}$$
$$H_{yr} = -\frac{\Gamma_0 E_0}{Z_0} e^{j\omega t+\gamma_0 z} \qquad (3.5)$$

Hence the total fields in region 1 are given, from (3.4) and (3.5) by

$$E_{1x} = E_0 e^{j\omega t}\left[e^{-\gamma_0 z} + \Gamma_0 e^{\gamma_0 z}\right]$$
$$H_{1y} = \frac{E_0}{Z_0} e^{j\omega t}\left[e^{-\gamma_0 z} - \Gamma_0 e^{\gamma_0 z}\right] \qquad (3.6a)$$

Similarly, the waves in the plasma (region 2) can be expressed as

$$E_{2x} = E_1 e^{j\omega t}\left[e^{-\gamma z} + \Gamma_1 e^{\gamma z}\right]$$
$$H_{2y} = \frac{E_1}{Z} e^{j\omega t}\left[e^{-\gamma z} - \Gamma_1 e^{\gamma z}\right] \qquad (3.6b)$$

where: Γ_1 is the amplitude reflection coefficient for the waves incident at boundary 2-3 and

$$Z = \frac{1}{\sqrt{\varepsilon_0 \mu_0 K}} = \frac{Z_0}{\sqrt{K}} \qquad (3.7a)$$

Z is the plasma characteristic impedance where K is the dielectric coefficient of the plasma. The permeability, μ, of the plasma is assumed equal to unity. Further

$$\gamma = jkK^{1/2} = \alpha + j\beta \tag{3.7b}$$

γ is the propagation constant of the waves in the plasma. α and β are functions of the dielectric coefficient.

The wave travelling from left to right in region 3 is given by

$$\begin{aligned} E_{3x} &= T_0 E_0 e^{j\omega t} e^{-\gamma_0 z} \\ H_{3y} &= T_0 \frac{E_0}{Z_0} e^{j\omega t} e^{-\gamma_0 z} \end{aligned} \tag{3.8}$$

where: T_0 is tho amplitude transmission coefficient for the waves incident on boundary 1 and transmitted through boundary 3.

Now matching the fields at boundary 1-2 ($z = 0$) gives, from Eqns. (3.6a) and (3.6b)

$$\begin{aligned} E_0(1 + \Gamma_0) &= E_1(1 + \Gamma_1) \\ \frac{E_0}{Z_0}(1 - \Gamma_0) &= \frac{E_1}{Z}(1 - \Gamma_1) \end{aligned} \tag{3.9}$$

At boundary 2-3 ($z = d$) from (3.6b) and (3.8) it is found that

$$\begin{aligned} E_1(e^{-\gamma d} + \Gamma_1 e^{\gamma d}) &= T_0 E_0 e^{-\gamma_0 d} \\ \frac{E_1}{Z}(e^{-\gamma d} - \Gamma_1 e^{\gamma d}) &= \frac{T_0 E_0}{Z_0} e^{-\gamma_0 d} \end{aligned} \tag{3.10}$$

Solving Eqns. (3.9) and (3.10) for T_0 and Γ_0 as a function of Z and γd yields

$$T_0 e^{-\gamma_0 d} = \left[\cosh \gamma d + \frac{1}{2}\left(\frac{Z}{Z_0} + \frac{Z_0}{Z}\right)\sinh \gamma d\right]^{-1} \tag{3.11}$$

$$\Gamma_0 = \frac{\frac{1}{2}\left(\frac{Z}{Z_0} - \frac{Z_0}{Z}\right)\sinh \gamma d}{\cosh \gamma d + \frac{1}{2}\left(\frac{Z}{Z_0} + \frac{Z_0}{Z}\right)\sinh \gamma d} \tag{3.12}$$

From Eqns. (3.7a) and (3.7b), the ratios of the characteristic impedance of the plasma and of free space are:

$$\frac{Z}{Z_0} = \frac{\beta}{k} - j\,\frac{\alpha}{k} \tag{3.13}$$

$$\frac{Z_0}{Z} = \frac{k(\beta + j\alpha)}{\alpha^2 + \beta^2} \tag{3.14}$$

The factors in Eqns. (3.11) and (3.12) involving the characteristic impedance can be replaced by expressions which are functions of the propagation parameters by the following substitutions:

$$\frac{Z}{Z_0} - \frac{Z_0}{Z} = 2(L + jM) \tag{3.15}$$

and

$$\frac{Z}{Z_0} + \frac{Z_0}{Z} = 2(P + jQ) \tag{3.16}$$

where

$$P = \frac{\beta}{2k}\,\frac{(k^2 + \alpha^2 + \beta^2)}{\alpha^2 + \beta^2} \qquad Q = \frac{\alpha}{2k}\,\frac{(k^2 - \alpha^2 - \beta^2)}{\alpha^2 + \beta^2}$$

$$L = \frac{\beta}{2k}\,\frac{(k^2 - \alpha^2 - \beta^2)}{\alpha^2 + \beta^2} \qquad M = \frac{\alpha}{2k}\,\frac{(k^2 + \alpha^2 + \beta^2)}{\alpha^2 + \beta^2}$$

The power transmissivity is given by the product of the amplitude transmission coefficient and its complex conjugate. Equations (3.11), and (3.16) yield after some manipulation

$$|T|^2 = Te^{-\gamma_0 d} \cdot T^* \cdot e^{-\gamma_0^* d} = T \cdot T^* \tag{3.17}$$

$$= \frac{1}{(P^2+Q^2)(\sinh^2\alpha d + \sin^2\beta d) + 2P\cosh\alpha d \sinh\alpha d + \sinh^2\alpha d + \cos^2\beta d - 2Q\sin\beta d\cos\beta d}$$

Similarly from (3.12) (3.15) and (3.16) the expression for the reflectivity is given by

$$|\Gamma|^2 = \frac{(L^2+M^2)(\sinh^2\alpha d + \sin^2\beta d)}{(P^2+Q^2)(\sinh^2\alpha d + \sin^2\beta d) + 2P\cosh\alpha d\sinh\alpha d - 2Q\sin\beta d\cos\beta d + \sinh^2\alpha d + \cos^2\beta d} \tag{3.18}$$

which leads, from Eqn.(3.3), to the relation for the absorptivity

$$A_\omega = 1 - \frac{1 + (L^2+M^2)(\sinh^2\alpha d + \sin^2\beta d)}{(P^2+Q^2)(\sinh^2\alpha d + \sin^2\beta d) + 2P\cosh\alpha d\sinh\alpha d + \sinh^2\alpha d + \cos^2\beta d - 2Q\sin\beta d\cos\beta d} \tag{3.19}$$

This relation gives the absorptivity of a uniform plasma slab for an incident plane electromagnetic wave in terms of the electromagnetic properties and thickness of the plasma.

Eqn.(3.19) is cumbersome to evaluate. However, simple approximations are possible for both high and low r-f frequencies relative to the plasma frequency and electron cyclotron frequency (i.e. $\omega \gtrless \omega_p$, $\omega \gtrless \omega_b$).

(a) <u>High Frequency Approximation</u>

At high frequencies where $(\omega/\nu)^2 >> 1$ and $(\omega/\omega_p)^2 >> 1$, $(\omega/\omega_b)^2 >> 1$ and where ω_p is of the same order of magnitude as ω_b, the expression for the absorptivity is

$$A_\omega \cong 1 - \frac{1}{1 + 2\alpha d + 2(\alpha d)^2} \tag{3.20}$$

Further, for a tenuous plasma

$$A_\omega \cong 2\alpha d \cong \frac{\nu}{c}\, d\,(\omega_p/\omega)^2 \qquad (\alpha d << 1) \tag{3.21}$$

For an opaque plasma

$$A_\omega \cong \frac{2\sqrt{2\omega\nu}}{\omega_p + \sqrt{2\omega\nu}} \qquad (\alpha d >> 1) \qquad (3.22)$$

The intermediate steps for these expressions are tabulated in Table 3.1. Note that for the tenuous plasma the absorptivity is directly proportional to the thickness of the slab and the collision frequency and inversely proportional to frequency squared. This implies that the radiation spectrum (Eqn. 3.1) is independent of frequency in the regions where self-absorption is negligible, as predicted by Dellis[13]. In addition, it should be noticed that in the high frequency limit the plasma is essentially isotropic, and the expression for A_ω is essentially independent of orientation of magnetic field.

(b) Low Frequency Approximations

At low frequencies the plasma remains anisotropic and hence the absorptivity at different directions to the magnetic field does not lead to the same value as was the case for the high frequency region. The absorptivity as given by equation (3.14) using appropriate approximations is listed below. The intermediate steps are tabulated in Table 3.1.

The low frequency assumptions are

$$\frac{\omega_b}{\omega} >> 1 \quad \left(\frac{\omega_p}{\omega}\right)^2 >> 1 \quad \left(\frac{\nu}{\omega}\right)^2 >> 1$$

ω_p and ω_b are of the same order of magnitude.

(1) Absorptivity parallel to the magnetic field.

(a) Electron Cyclotron Wave. When the plasma is thin and appears tenuous to this wave

$$A_\omega \cong \frac{\nu/\omega_p}{\frac{\omega_p d}{4c} + 4\left(\frac{\omega_b}{\omega_p}\right)^2 \frac{c}{d\omega_p} + \frac{\nu}{\omega_p}} \qquad (\alpha d << 1) \qquad (3.23)$$

Wave	K_r	K_i	(α/k)	(β/k)	αd	βd	$A_\omega(\alpha d << 1)$	$A_\omega(\alpha d >> 1)$	Conditions
				HIGH FREQUENCIES					
All Waves	$1 - \left(\frac{\omega_p}{\omega}\right)^2$	$\left(\frac{\omega_p}{\omega}\right)^2 \frac{\nu}{\omega}$	$\frac{1}{2}\frac{\nu}{\omega}\left(\frac{\omega_p}{\omega}\right)^2$	$1 - \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^2$	$\frac{1}{2}\frac{\nu}{c} d \left(\frac{\omega_p}{\omega}\right)^2$	$\frac{\omega d}{c}$	$\frac{\nu}{c} d \left(\frac{\omega_p}{\omega}\right)^2$	1	$\omega^2 >> \omega_p^2$ $\omega^2 >> \omega_b^2$ $\omega^2 >> \nu^2$
				LOW FREQUENCIES					
Electron Cyclotron Wave	$\frac{\omega_p^2}{\omega\omega_b}$	$\left(\frac{\omega_p}{\omega_b}\right)^2 \frac{\nu}{\omega}$	$\frac{\omega_p \nu}{2\omega_b\sqrt{\omega\omega_b}}$	$\frac{\omega_p}{\sqrt{\omega\omega_b}}$	$\sqrt{\frac{\omega}{\omega_b}}\,\frac{\omega_p \nu d}{2\omega_b c}$	$\sqrt{\frac{\omega}{\omega_b}}\,\frac{\omega_p d}{c}$	$\frac{\nu/\omega_p}{\frac{\omega_p d}{4c} + 4\left(\frac{\omega_b}{\omega_p}\right)^2 \frac{c}{d\omega_p} + \frac{\nu}{\omega_p}}$	$\frac{4\omega_p}{\sqrt{\omega\omega_b}\left(1 + \frac{\omega_p}{\sqrt{\omega\omega_b}}\right)^2}$	$\omega^2 << \nu^2$ $\omega^2 << \omega_p^2$ $\nu < \omega_p$ $\omega_p \sim \omega_b$
Ion Cyclotron Wave	$-\frac{\omega_p^2}{\omega\omega_b}$	$\left(\frac{\omega_p}{\omega_b}\right)^2 \frac{\nu}{\omega}$	$\frac{\omega_p}{\sqrt{\omega\omega_b}}$	$\left(\frac{\omega_p}{\omega_b}\right)^2 \frac{\nu}{\omega}$	$\sqrt{\frac{\omega}{\omega_b}}\,\frac{\omega_p d}{c}$	$\sqrt{\frac{\omega}{\omega_b}}\,\frac{\omega_p \nu d}{2\omega_b c}$	$\frac{\nu/\omega_p}{\frac{\omega_p d}{4c} + 4\left(\frac{\omega_b}{\omega_p}\right)^2 \frac{c}{d\omega_p} + \frac{\nu}{\omega_p}}$	$2\sqrt{\frac{\omega}{\omega_b}}\,\frac{\nu}{\omega_p}$	$\omega^2 << \nu^2$ $\omega^2 << \omega_p^2$ $\nu < \omega_p$ $\omega_p \sim \omega_b$
Ordinary Wave	$-\left(\frac{\omega_p}{\nu}\right)^2$	$\frac{\omega_p^2}{\omega\nu}$	$\frac{\omega_p}{\sqrt{2\omega\nu}}$	$\frac{\omega_p}{\sqrt{2\omega\nu}}$	$\sqrt{\frac{\omega}{2\nu}}\,\frac{\omega_p d}{c}$	$\sqrt{\frac{\omega}{2\nu}}\,\frac{\omega_p d}{c}$	$\frac{\omega_p^2 d/\nu c}{\left[1 + \frac{\omega_p^2 d}{2\nu c}\right]^2}$	$\frac{2\sqrt{2\omega\nu}}{\omega_p + \sqrt{2\omega\nu}}$	$\omega^2 << \nu^2$ $\omega^2 << \omega_p^2$ $\nu < \omega_p$
Extra-Ordinary Wave	$-\left(\frac{\omega_p}{\nu}\right)^2 \left(1 + \left(\frac{\omega_b}{\omega_p}\right)^2\right)$	$\frac{\omega_p^2}{\omega\nu}$	$\frac{\omega_p}{\sqrt{2\omega\nu}}$	$\frac{\omega_p}{\sqrt{2\omega\nu}}$	$\sqrt{\frac{\omega}{2\nu}}\,\frac{\omega_p d}{c}$	$\sqrt{\frac{\omega}{2\nu}}\,\frac{\omega_p d}{c}$	$\frac{\omega_p^2 d/c\nu}{\left[1 + \frac{\omega_p d}{2\nu c}\right]^2}$	$\frac{2\sqrt{2\omega\nu}}{\omega_p + \sqrt{2\omega\nu}}$	$\omega^2 << \nu^2$ $\omega^2 << \omega_p^2$ $\nu < \omega_p$ $\omega_p \sim \omega_b$

Table 3.1. Approximations for the propagation parameters and absorptivity of a uniform anisotropic plasma.

Note that the absorptivity for very low frequencies is independent of frequency (when $\alpha d \ll 1$) for this wave. As will be shown this is also true for the other waves considered here.

When the plasma is very thick and the condition $\alpha d \gg 1$ holds,

$$A_\omega \cong \frac{4\omega_p}{\sqrt{\omega\omega_b}\left(1 + \dfrac{\omega_p}{\sqrt{\omega\omega_b}}\right)^2} \qquad (\alpha d \gg 1) \qquad (3.24)$$

From Table 3.1 it is clear that the attenuation coefficient, α, is small so that physically Eqn. (3.24) represents the case of a very thick slab. It can be seen that as the frequency ω tends to zero A_ω tends to zero as $\omega^{1/2}$. This result is somewhat surprising since it indicates that at very low frequencies the absorptivity of a thin slab may be larger than that for a thick one. The absorptivity of a very thick plasma ($\alpha d \gg 1$) is independent of d since all the energy which penetrates the first boundary is absorbed. These remarks also apply to the other waves as will be shown.

(b) Ion Cyclotron Wave. When the plasma appears tenuous

$$A_\omega = \frac{\nu/\omega_p}{\dfrac{\omega_p d}{4c} + 4\left(\dfrac{\omega_b}{\omega_p}\right)^2 \dfrac{c}{d\omega_p} + \dfrac{\nu}{\omega_p}} \qquad (\alpha d \ll 1) \qquad (3.25)$$

This is identical with that for the electron cyclotron wave. When the plasma appears opaque

$$A_\omega \cong 2\sqrt{\omega/\omega_b} \cdot \nu/\omega_p \qquad (\alpha d \gg 1) \qquad (3.26)$$

The absorptivity for the ion cyclotron wave (neglecting ion effects) also decreases as $\omega^{1/2}$ and in this case is directly proportional to the electron collision frequency.

(2) Absorptivity transverse to the magnetic field.

(a) Ordinary Wave. When the plasma appears tenuous

$$A_\omega \cong \frac{2(\alpha/k)(\alpha d)}{[1 + (\alpha/k)(\alpha d)]^2} \cong \frac{\omega_p^2 d/\nu c}{\left[1 + \frac{\omega_p^2 d}{2\nu c}\right]^2} \qquad (\alpha d << 1) \qquad (3.27)$$

Clearly, the absorptivity is again independent of frequency and if considered as a function of d has a maximum for $d = \frac{2\nu c}{\omega_p^2}$.

When the plasma is opaque

$$A_\omega \cong \frac{1 - \frac{1}{2}(\alpha/k)^2}{1 + (\alpha/k) + \frac{1}{2}(\alpha/k)^2} \qquad (\alpha d >> 1) \qquad (3.28)$$

$$A\omega \cong \frac{2\sqrt{\omega\nu}}{\omega_p + \sqrt{2\omega\nu}}$$

The absorptivity again decreases as the frequency is lowered.

(b) Extraordinary Wave. When the plasma appears tenuous

$$A_\omega \cong \frac{\omega_p^2 d}{\left[1 + \frac{\omega_p^2 d}{2\nu c}\right]^2} \qquad (\alpha d << 1) \qquad (3.29)$$

and

$$A\omega \cong \frac{2\sqrt{2\omega\nu}}{\omega_p + 2\sqrt{\omega\nu}} \qquad (\alpha d >> 1) \qquad (3.30)$$

These two expressions are identical to those obtained for the ordinary wave.

(c) Effect of Boundaries on the Absorptivity Spectrum.

If the free-space—plasma interface has a very small reflection coefficient, implying a good match between free space and the plasma, the absorptivity, is simply unity less the transmissivity (Eqn. 3.3). If, on the other hand, the reflection coefficient is large there is a bad mismatch and the absorptivity is always small since little energy can penetrate the plasma.

Another effect which is likely to modify the absorptivity spectrum is that due to multiple internal reflections of the energy between the boundaries of the slab. If one examines the general solution Eqn.(3.19) for A_ω it is noted that for a continuous change in the slab thickness, d, the functions $\sin\beta d$ and $\cos\beta d$ will go through minimum and maximum values every time βd goes through an integral number of $\pi/2$. Thus, it is expected that these functions will give rise to undulations in the value of absorptivity as function of the slab thickness. A similar effect will take place if instead of varying the slab thickness one studies the frequency spectrum. In this case as ω is varied β also changes and βd may become equal to an integral number of $\pi/2$. This effect may be examined in a more quantitative manner as follows.

Consider the case where $\beta d = n\pi$ (n an integer) and let A_π denote the absorptivity where $\beta d = n\pi$. Eqn. (3.19) becomes

$$A_\pi = \frac{[P^2 + Q^2 - L^2 - M^2 + 1]\ \tanh^2\alpha d + 2P\ \tanh\alpha d}{(P^2 + Q^2)\ \tanh^2\alpha d + 2P\ \tanh\alpha d + 1} \tag{3.31}$$

If $(\beta/k)^2 >> (\alpha/k)^2$ and if (β/k) is not close to unity undulations in the spectrum are to be expected as long as αd is less than unity, since then the wave can be reflected many times before being completely attenuated. Equation (3.31) then reduces to

$$A_\pi \cong \frac{2\tanh\alpha d\,(P + \tanh\alpha d\,)}{(P\tanh\alpha d + 1)^2} \tag{3.32}$$

Consider next the case where βd $(n + \frac{1}{2})\pi$ (n an integer). Let $A_{\frac{\pi}{2}}$ be the absorptivity where this condition holds. For this case

$$A_{\frac{\pi}{2}} = \frac{(P^2 + Q^2 - L^2 - M^2) + 2P \tanh \alpha d + 2\tanh^2 \alpha d - 1}{(P^2 + Q^2) + 2P \tanh \alpha d + \tanh^2 \alpha d} \tag{3.33}$$

Again, if $(\beta/k)^2 >> (\alpha/k)^2$ Eqn. (3.33) becomes

$$A_{\frac{\pi}{2}} \cong \frac{2\sinh \alpha d}{(P\cosh \alpha d + \sinh \alpha d)} = \frac{2\tanh \alpha d}{P + \tanh \alpha d} \tag{3.34}$$

An estimate of the amplitude of the undulations may be made by considering the ratio of $A\pi$ to $A\pi/2$. Assuming that αd and P are the same value for both $A\pi$ and $A\pi/2$, then from Eqns.(3.32) and (3.33)

$$A_{\pi}/A_{\frac{\pi}{2}} = \frac{(P + \tanh \alpha d)^2}{(P\tanh \alpha d + 1)^2} \tag{3.35}$$

$$P \cong \frac{1 + (\beta/k)^2}{2\beta/k}$$

Since $(1 - \beta/k)^2$ is always greater than or equal to zero

$$1 + (\beta/k)^2 \geq 2(\beta/k)$$

Consequently, P is always ≥ 1.

Since $\tanh \alpha d$ is never greater than unity then $A_{\pi}/A_{\frac{\pi}{2}} \geq 1$. The absorptivity spectrum will, therefore, go through a maximum when $\beta d = n\pi$ and a minimum when $\beta d = (n + \frac{1}{2})\pi$. The amplitude of the undulations is a function of (β/k) and αd.

3.3 The Absorptivity Spectrum

The absorptivity spectrum has been computed (using Eqn. 3.19) as a function of normalized frequency (ω/ω_p) for a uniform d-c magnetic field both along the direction of propagation and transverse to the direction

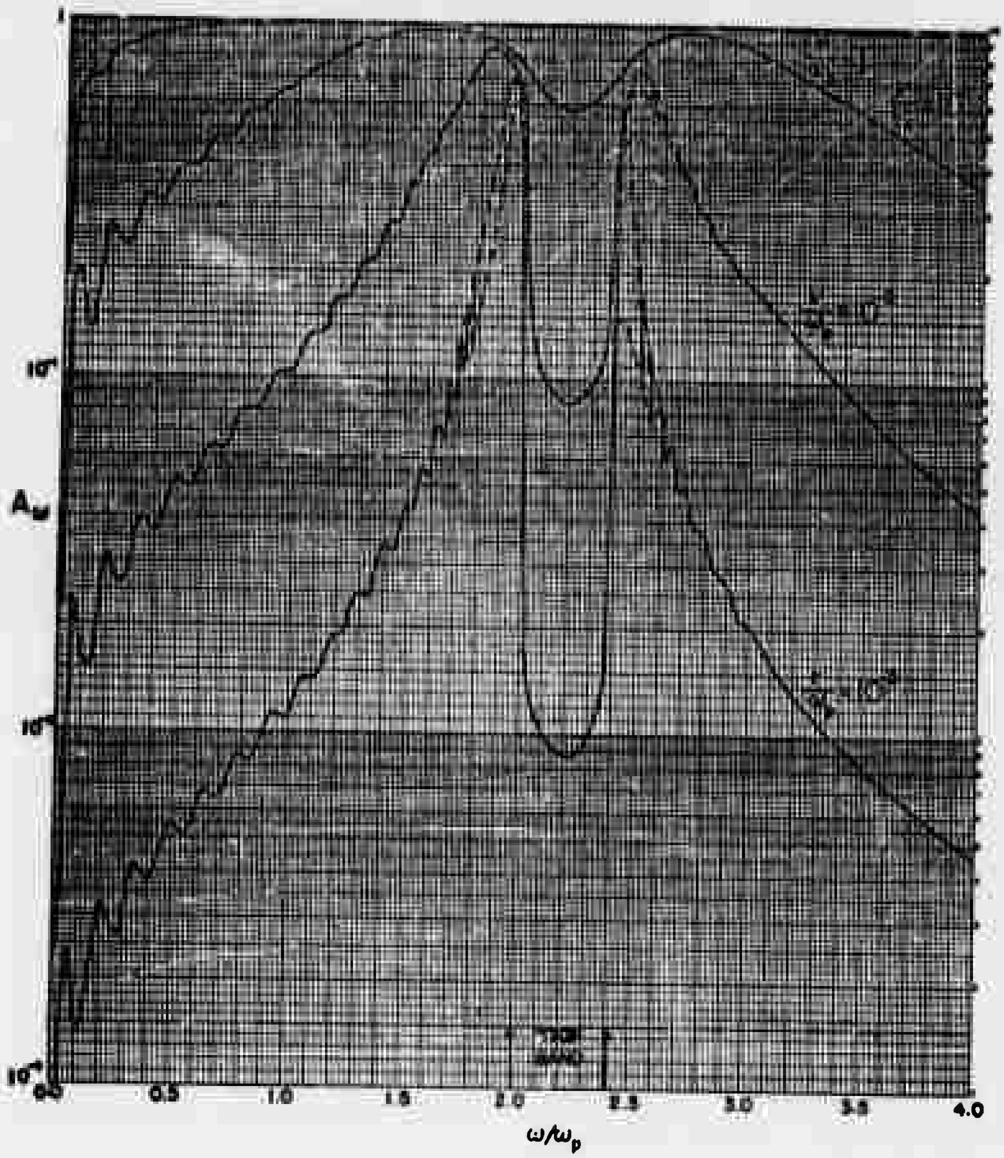

Fig. 3.2 Absorptivity spectrum of electron cyclotron wave for various electron collision frequencies. $\left(\frac{\omega_b}{\omega_p} = 2\right)$

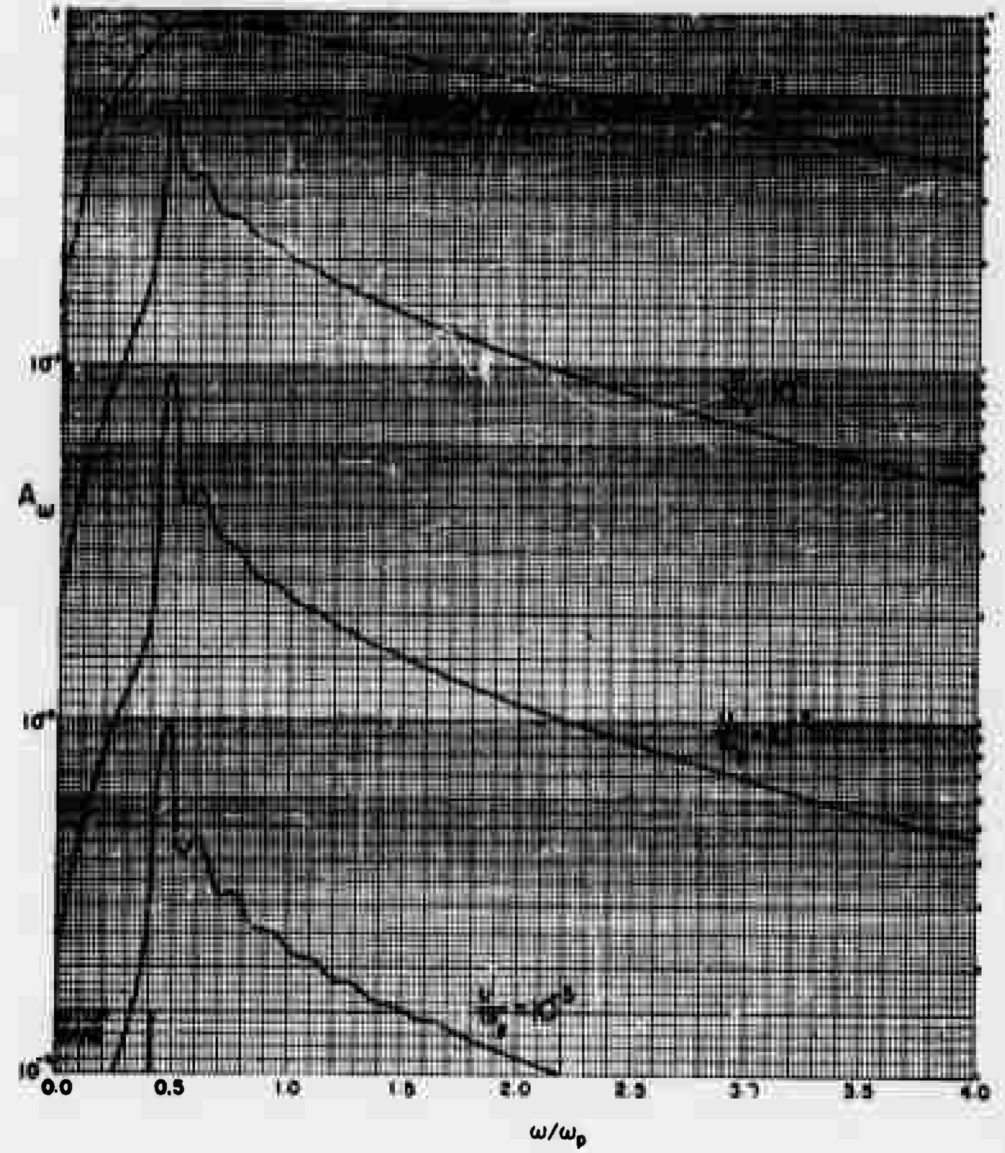

Fig. 3.3 Absorptivity spectrum of ion cyclotron wave for various electron collision frequencies. ($\omega_b/\omega_p = 2$).

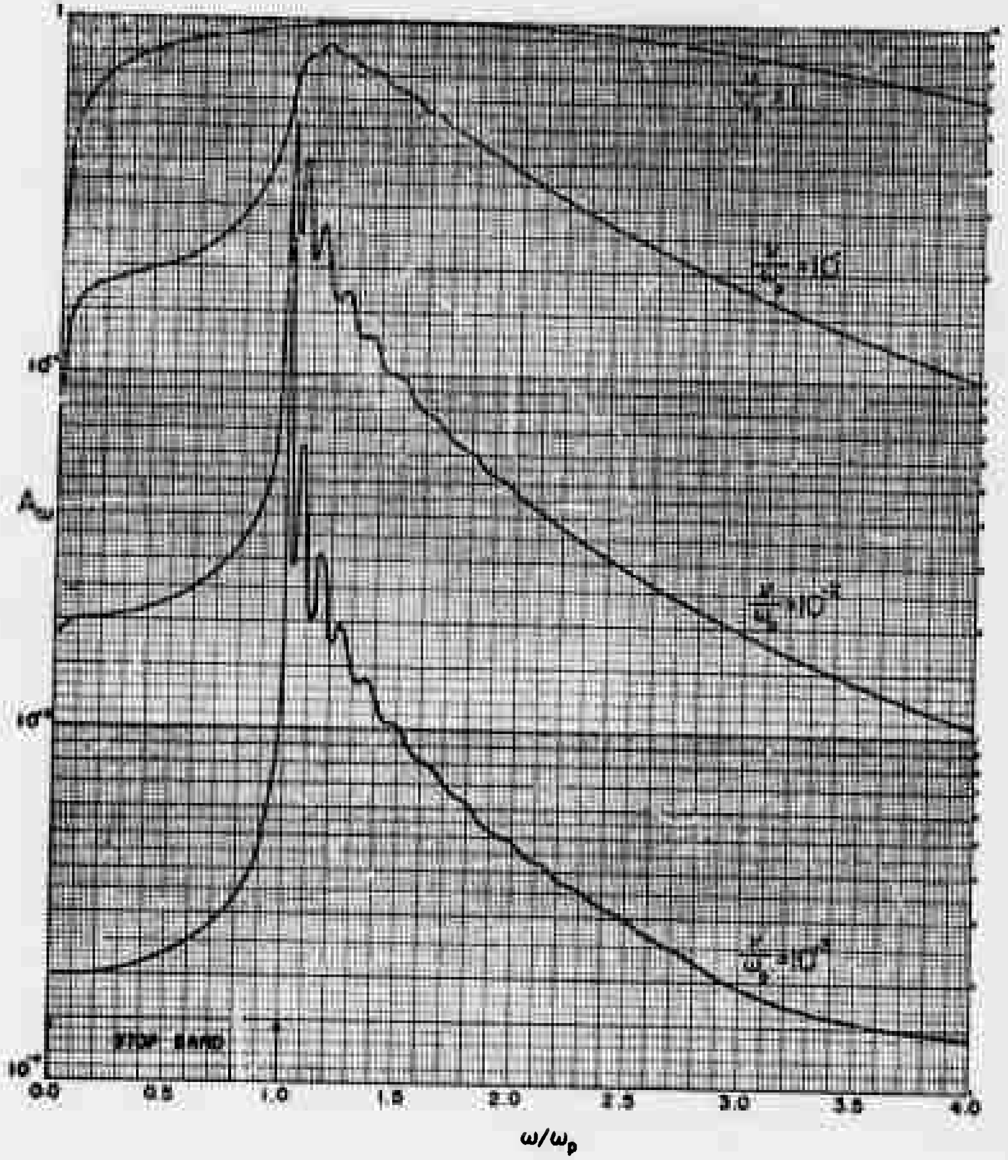

Fig. 3.4 Absorptivity spectrum of ordinary wave for various electron collision frequencies.

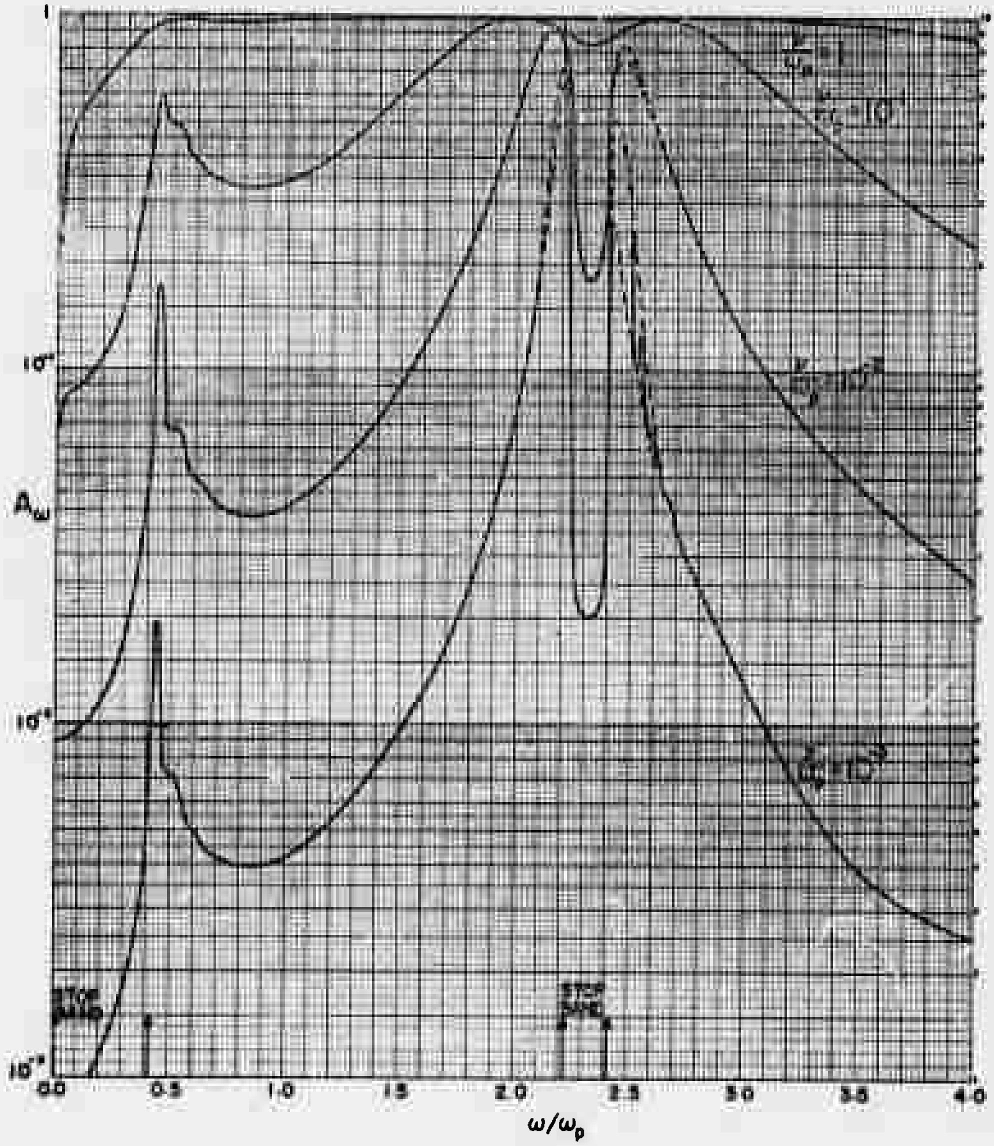

Fig. 3.5 Absorptivity spectrum of extraordinary wave for various electron collision frequencies. $(\omega_b/\omega_p = 2)$.

of propagation. Four spectra result, corresponding to the four possible waves discussed earlier; the dielectric coefficient for each wave being listed in Table 2.2. The effect of electron density and collision frequency on the absorptivity was investigated by computing a set of curves for a fixed plasma thickness ($d/\lambda_p = 2.5$) and values of ν/ω_p ranging from 10^{-3} to 1 (Figs. 3.2 - 3.5). The variation of absorptivity with plasma thickness was for a value of $\nu/\omega_p = 10^{-1}$ and values of d/λ_p ranging from 0.05 to ∞ (Figs. 3.6 - 3.9). In all cases a constant value of $\omega_b/\omega_p = 2$ was chosen. (λ_p is defined as the wavelength corresponding to the plasma frequency and is hence a measure of electron density in the plasma).

The main characteristic of the results is that the absorptivities have a maximum just outside the edges of the stop-bands (as indicated) except for very high collision frequencies ($\nu \geq \omega_p$) and very low thickness ($d \ll \lambda_p$). Another characteristic is the presence of undulations in the absorptivity spectrum. These undulations are caused by internal reflections from the slab boundaries. A further characteristic is that the absorptivity of a very thin slab is often greater than that of an infinite slab at very low frequencies. This effect, which cannot be predicted on the basis of the geometrical optics theory, seems to appear as a result of a solution based on a more exact boundary value treatment. In any case, for a plasma slab small compared with the wavelength of the radiation the concept of ray tracing cannot be applied.

In any practical case the total absorptivity of an anisotropic plasma slab in a direction normal to its boundaries is a combination of the absorptivities of the two possible waves in this direction. For example, if the externally applied magnetic field B_0 is along the direction of propagation (Fig. 3.1a) the absorptivities of the electron (A_e) and ion (A_i) cyclotron waves must be combined.

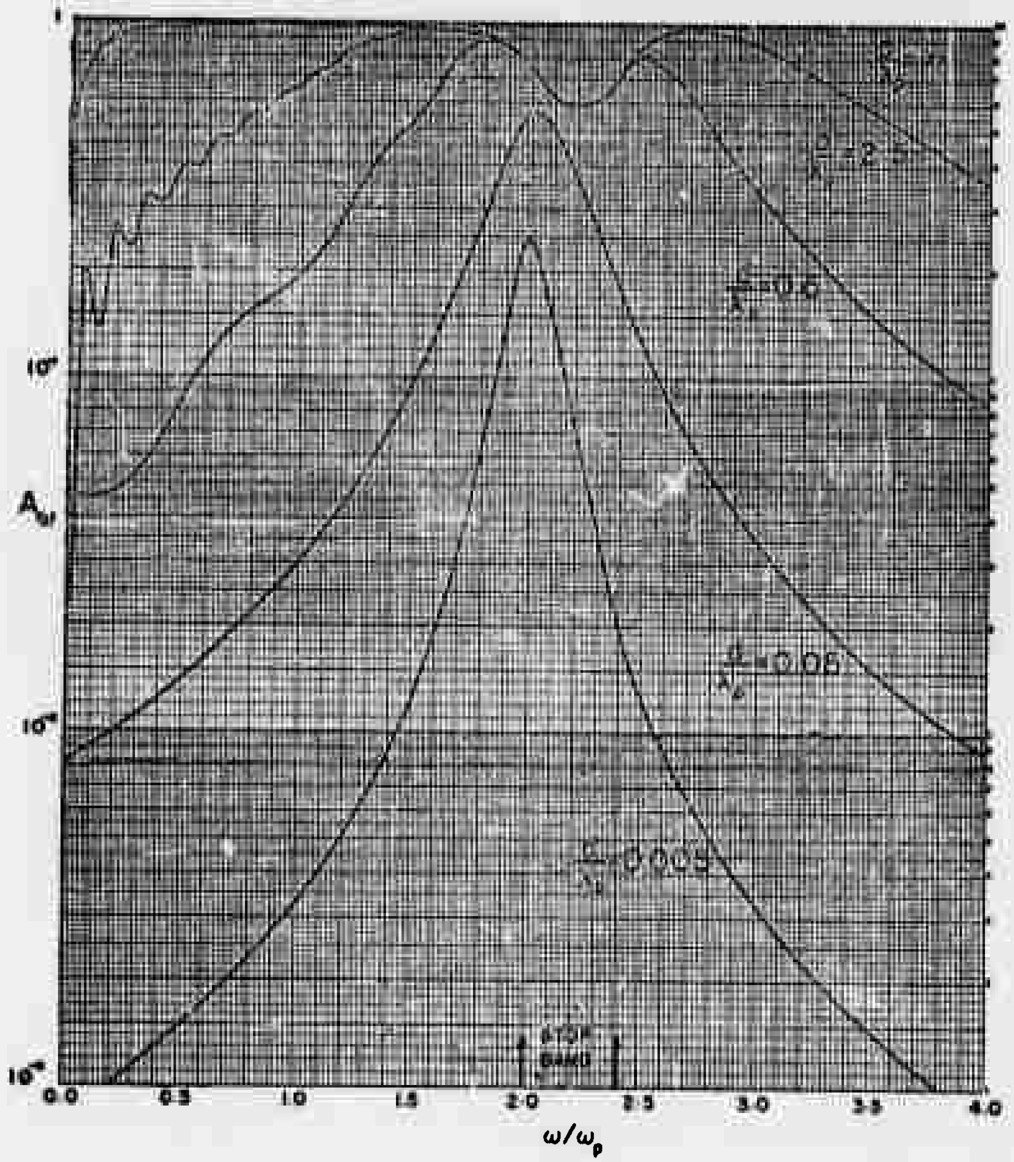

Fig. 3.6 Absorptivity spectrum of electron cyclotron wave for various slab thicknesses. $\left(\frac{\omega_b}{\omega_p} = 2\right)$

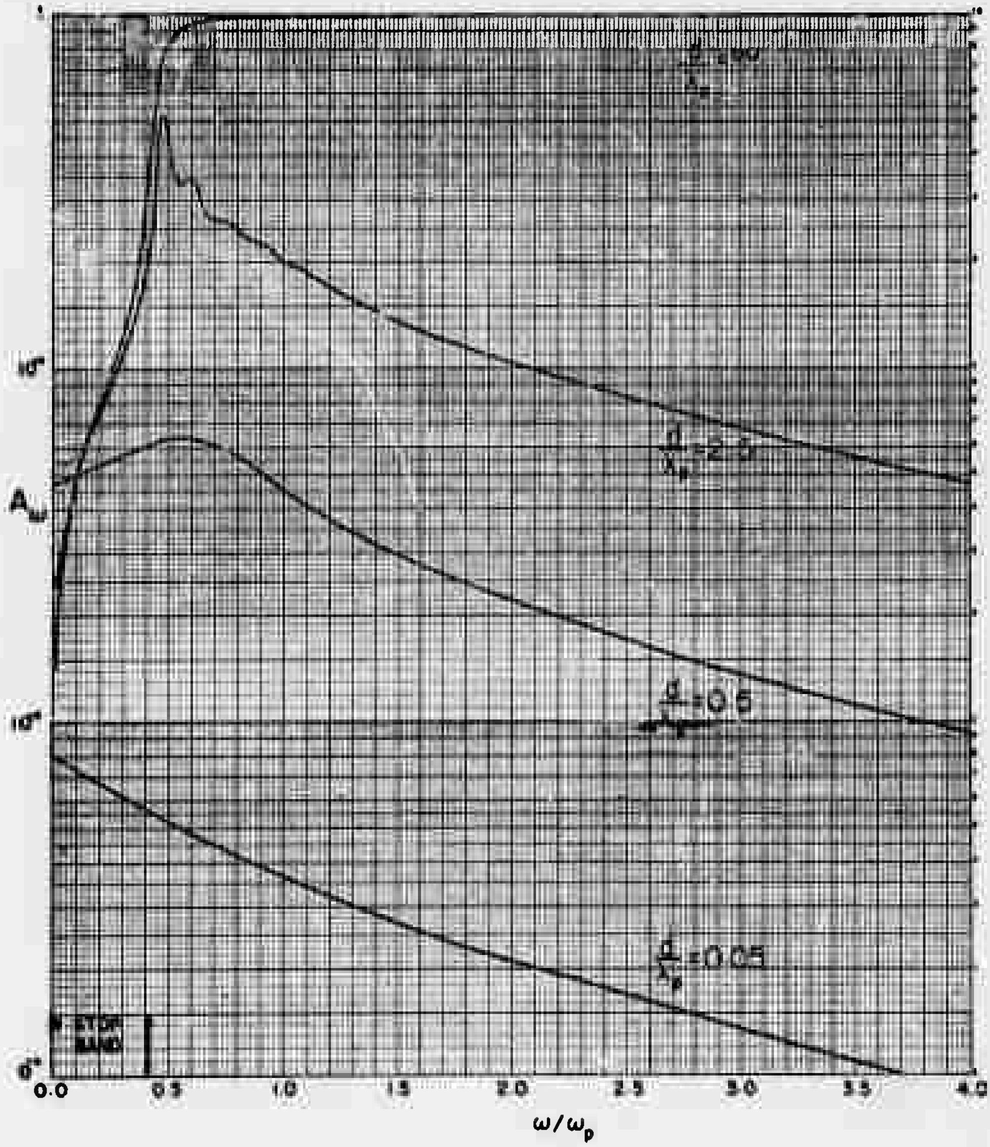

Fig. 3.7 Absorptivity spectrum of ion cyclotron wave for various slab thicknesses. $(\omega_b/\omega_p = 2)$.

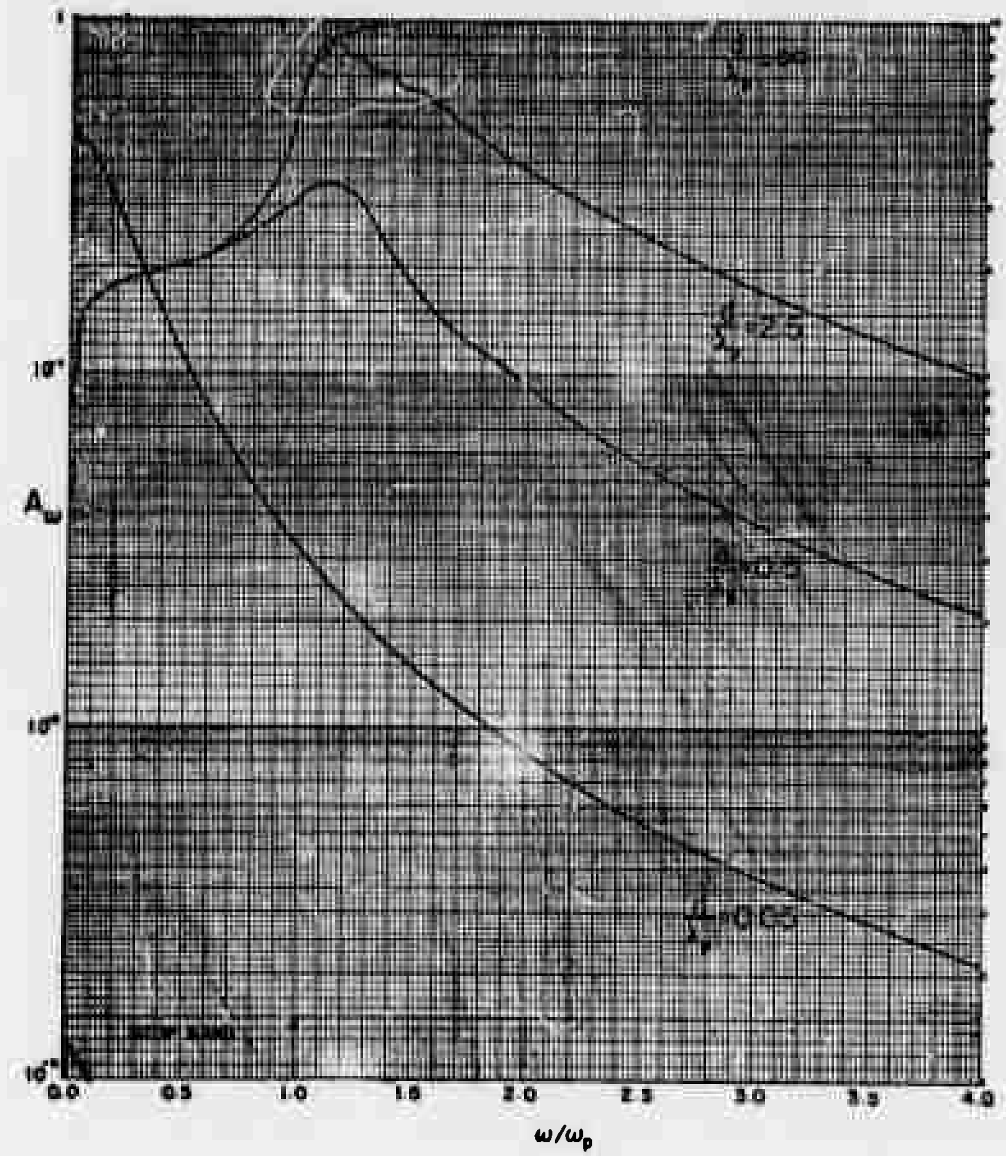

Fig. 3.8 Absorptivity spectrum of ordinary wave for various slab thicknesses.

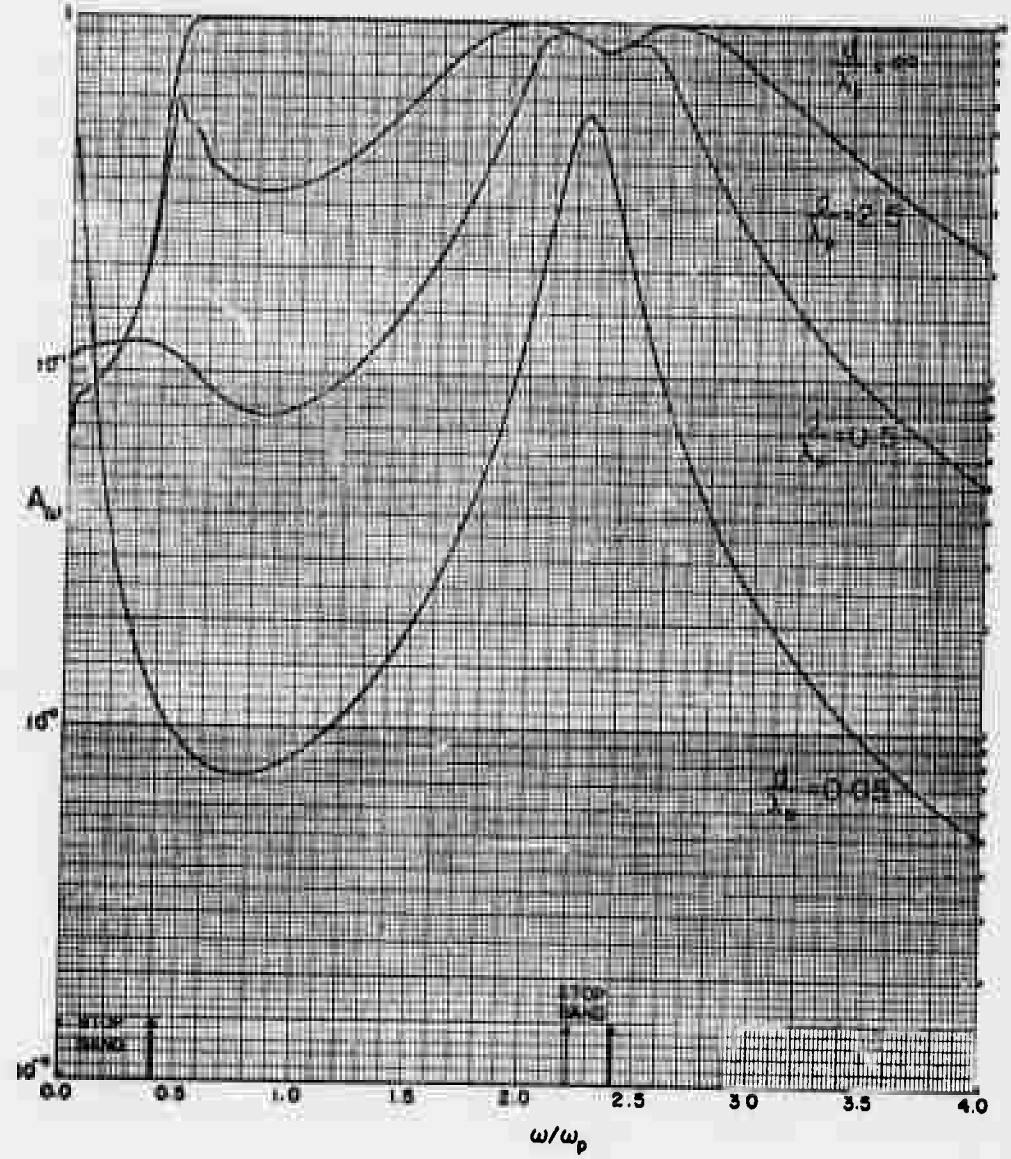

Fig. 3.9 Absorptivity spectrum of extraordinary wave for various slab thicknesses. $(\omega_b/\omega_p = 2)$.

The linearly polarized uniform plane wave carrying unit power considered in the first section can be regarded as two equal amplitude circularly polarized waves rotating in opposite directions. One of these circularly polarized waves is the electron cyclotron and the other the ion cyclotron wave. Since each wave carries half the power the absorptivity is the mean of A_e and A_i, i.e.

$$A = \tfrac{1}{2}(A_e + A_i)$$

Similarly, when the applied magnetic field is perpendicular to the direction of propagation the total absorptivity is the mean of the absorptivity of the ordinary wave and extraordinary waves if a randomly polarized incident wave is used.

(a) Variations in absorptivity spectrum

(1) Low Absorptivity in Stop-Band

For a loss-less plasma ($\nu = 0$) the stop-bands are a region where the phase constant (β/k) becomes zero implying that no propagation can occur. The term "stop-band" for a plasma which has a small but finite number of collisions ($\nu << \omega_p$) implies a region where the normalized phase constant (β/k) drops very rapidly. It has little meaning when the collision frequency is high ($\nu \sim \omega_p$). Generally one expects enhanced absorptivity just outside a stop-band, since (β/k) is not very different from unity and most of the energy penetrates the first boundary and is absorbed due to the high attenuation constant, α.

In the stop-bands the plasma is badly matched to free-space (β/k not close to unity) resulting in a large reflection from the first boundary and low absorptivity since little energy enters the plasma.

(2) Effect of Electron Collision Frequency

In general, a high collision frequency implies a very lossy plasma and high absorptivity over the whole spectrum. A low collision frequency implies a bad mismatch to free space in the stop-bands and therefore very low absorptivity in this region. Over the rest of the spectrum the absorption path length, αd, is small and decreases as the electron collision frequency decreases, implying low absorption.

For example, at the centre of the stop-band $\beta/k = .06$ for the electron cyclotron wave when $\nu/\omega_p = 10^{-2}$. Decreasing the collision frequency by a factor of 10 to $\nu/\omega_p = 10^{-3}$ reduces β/k to .006 implying a much higher reflection.

The power reflection coefficient from the first boundary is given by[4]

$$\frac{(1-\beta/k)^2 + (\alpha/k)^2}{(1+\beta/k)^2 + (\alpha/k)^2}$$

The energy which does penetrate the first boundary is, however, all absorbed in its first passage through the slab since αd is very large ($\sim 100 - 200$) in the examples considered above.

(3) Effect of Slab Thickness

For an infinite plasma slab at frequencies greater than both the plasma frequency and the electron cyclotron frequency the absorptivity of a plasma increases as the slab thickness, until the absorptivity is practically unity. This means that the reflectivity is practically zero and the transmissivity is zero. For a thinner slab at these high frequencies the absorptivity decreases since the absorption path length, αd, decreases. At frequencies slightly below both the plasma and electron cyclotron resonances the absorptivity also increases with slab thickness. At very low frequencies, however, this theory gives some anomalous results. It appears that the absorptivity of a thin slab is very much greater than that of an infinite one. Also, the characteristic maxima at the stop-bands are modified or disappear.

(4) Effect of Electron Density and Magnetic Field

The effect of changing electron density is to change the plasma frequency in accordance with the relation

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m} \quad \text{or} \quad \lambda_p = \frac{2\pi c}{\omega_p}$$

The absorptivity curves (Figs. 3.2 to 3.9) are normalized with respect to plasma frequency, ω_p, or free-space wavelength, corresponding to the plasma frequency, λ_p, and are shown in terms of the parameters ν/ω_p, ω/ω_p and d/λ_p. A value of $\omega_b/\omega_p = 2$ was chosen.

In general, changing the magnetic field changes the position of the stop bands and the regions in the spectrum of enhanced radiation. For example, if the electron cyclotron frequency is made much larger than the plasma frequency then the stop-band for the electron cyclotron wave and the upper stop-band for the extra-ordinary wave becomes extremely narrow and their position almost coincides with the electron cyclotron frequency. Consequently, only a single peak about ω_b would appear in the absorptivity spectrum. A radiometer measuring radiation from such a plasma should measure a high intensity about ω_b and a very much smaller intensity due to the ordinary wave about ω_p. In such a case the effect of the ordinary and ion cyclotron waves on the total spectrum would be small.

Alternatively, making the plasma frequency much larger than the electron cyclotron frequency stretches out the stop-bands considerably. The stop band for ordinary, ion cyclotron and extraordinary waves extends effectively from zero to ω_p with a very narrow pass band for the extraordinary in the region of ω_p. The stop band for the electron cyclotron wave is from ω_b to ω_p. Consequently, enhanced absorptivity should be observed mainly around ω_p with a small peak around ω_b (if $\nu < \omega_b$) due to the lower edge of the electron cyclotron wave stop-band.

These remarks do not apply to a plasma whose collision frequency is high ($\nu \geq \omega_p$) since in this case stop-bands have no meaning.

The main point to note is that unless the electron cyclotron frequency is of the same order of magnitude as the plasma frequency the frequencies of enhanced absorptivity and radiation will be around ω_b and ω_p.

(b) Undulations in the Absorptivity Spectrum

(1) Position of Undulations

The effect of sharp plasma boundaries on the spectrum of absorptivity accounts for undulations which are sometimes quite violent. These undulations are due to internal reflections within the slab. It is clear from Fig. 3.10, in which the radiation spectrum (or absorptivity multiplied by frequency squared in arbitrary units) is shown for the ordinary wave (or an isotropic plasma), that peaks in the undulations occur when $\beta d = n\pi$ (n an integer) and dips when $\beta d = (n + \frac{1}{2})\pi$. The curve of $(\beta d/\pi)$ is shown for comparison. This result confirms the theoretical derivation presented in section 3.2 concerning the effect of internal reflections.

(2) Conditions When Undulations are Apparent

There are several conditions for enhancing the undulations. As discussed above, βd must be greater than π. The normalized phase constant

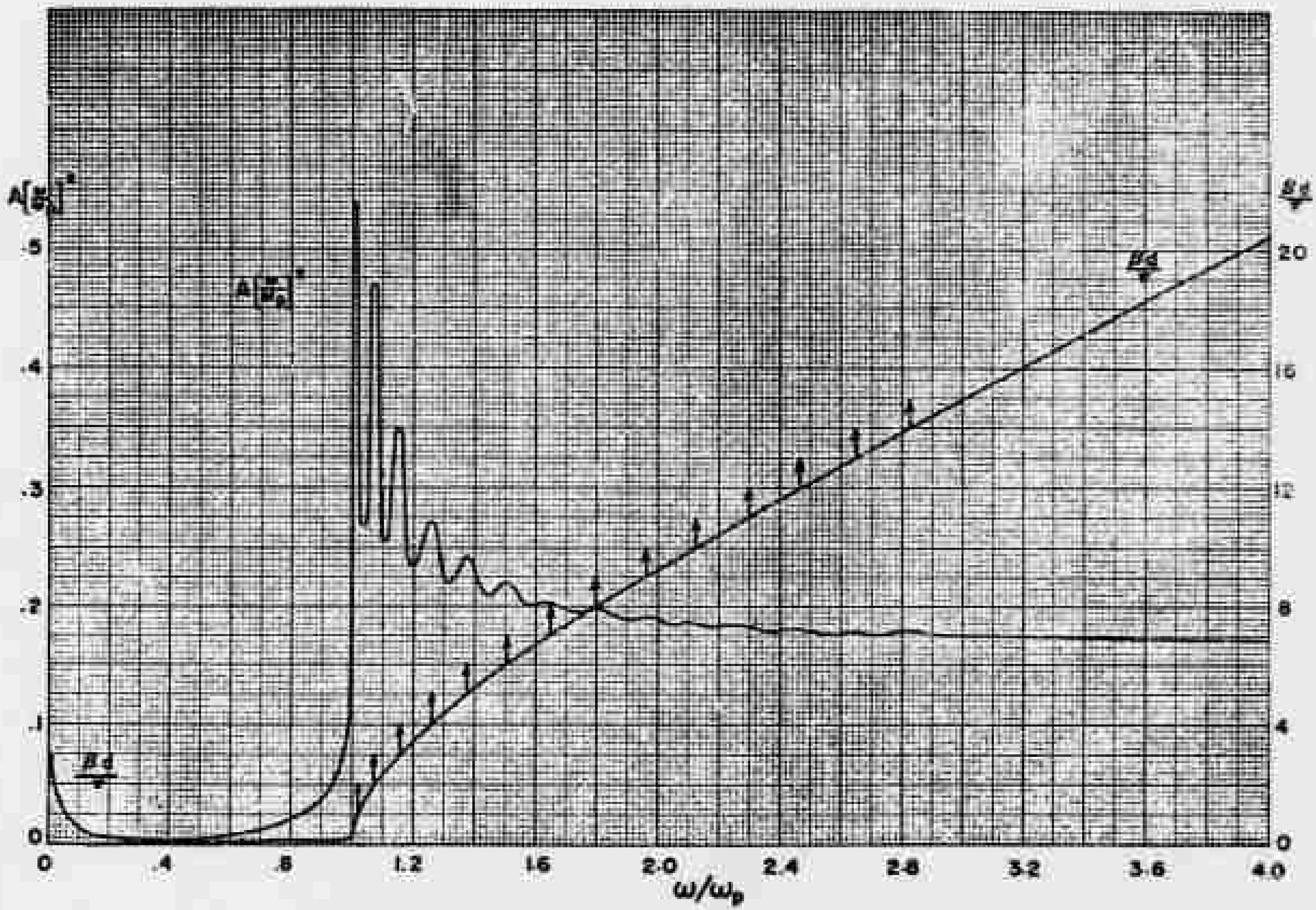

Fig. 3.10 Radiation spectrum (arbitrary units) for ordinary wave, $\left(\frac{\nu}{\omega_p} = 10^{-2},\ \frac{d}{\lambda_p} = 2.5\right)$ illustrating undulations due to wall effects.

(β/k) should not be close to unity to give a mismatch and hence large reflections. The absorption path length αd should be small (< 1), otherwise the wave will be highly attenuated in its first passage through the plasma and further reflections will be negligible.

For example, consider the variations for $\nu/\omega_p = 10^{-3}$ in Fig. 3.4. At:

$$\beta d = \pi, \qquad \omega = 1.01\ \omega_p$$

$$\beta/k \sim .1 \text{ and } \alpha d \sim .05$$

Consequently the wave can bounce back and forth many times before it is finally attenuated.

(3) <u>Effect of Electron Collision Frequency on the Undulations</u>

Undulations are always most violent when the collision frequency is small. This is due to the fact that at the absorption path length, αd, decreases with decreasing collision frequency. Consequently many internal reflections can occur depending on the plasma thickness and hence enhance the undulations in the absorptivity spectrum. The collision frequency has little effect on the phase constant and on the position of the undulations except when ν is close to the plasma frequency.

(4) <u>Effect of Slab Thickness on the Undulations</u>

The position of the undulations in the spectrum is proportional to βd and therefore to the thickness, d. If the thickness is very small the maximum value of βd may not reach $\pi/2$ and no undulations will be observed. A thick slab may have such a high αd that virtually all the energy is absorbed in its first passage through the plasma, thus damping out any undulations.

Fig. 3.11 shows the absorptivity as a function of slab thickness when $\omega = 1.2\omega_p$ for $\nu/\omega_p = 10^{-1}$ and 10^{-2}. The corresponding values of normalized attenuation constant, α/k, are .005 and .05, respectively. The undulations are much less pronounced for the higher collision frequency. They are completely damped out at large values of slab thickness. Fig. 3.12 shows the case when $\omega = \omega_p$, and corresponds to a much higher attenuation constant, α, and much lower phase constant, β. $\beta d = \pi$ corresponds to about $d/\lambda_p = 2.5$. Consequently, when the correct phase conditions for internal reflections occur, a high value of slab thickness and high attenuation constant damp out the undulations.

(c) Absorptivity at Low Frequencies

The results at low frequencies, previously discussed in connection with equations 3.23 to 3.30, where a thin slab may have a higher absorptivity than a very large one can not be anticipated from geometrical optics approximations. Geometrical optics ray tracing techniques are only valid when the ratio α/β is much smaller than unity[14]. Figs. 2.2 to 2.6 should be used to determine when geometrical optics approximations are valid. Generally α/β is much smaller than unity in the pass-bands, but not generally in the stop-bands, where it may be of the order of, or much larger than, unity. The theory used in section 2 is exact in all parts of the spectrum for the simple slab model used.

Fig. 3.13 illustrates the absorptivity as a function of slab thickness when $\omega = .1\omega_p$ for values of collision frequency corresponding to $\nu/\omega_p = 10^{-1}$ and 10^{-2}. The corresponding ratios of α/β for these two curves are 2 and 20, respectively. The maximum observed with these curves corresponds to the maximum already discussed in connection with equation (3.27). It should be noted that this maximum can explain the cross-over of the curves for different thicknesses at very low frequencies in Figs. 3.6 to 3.9.

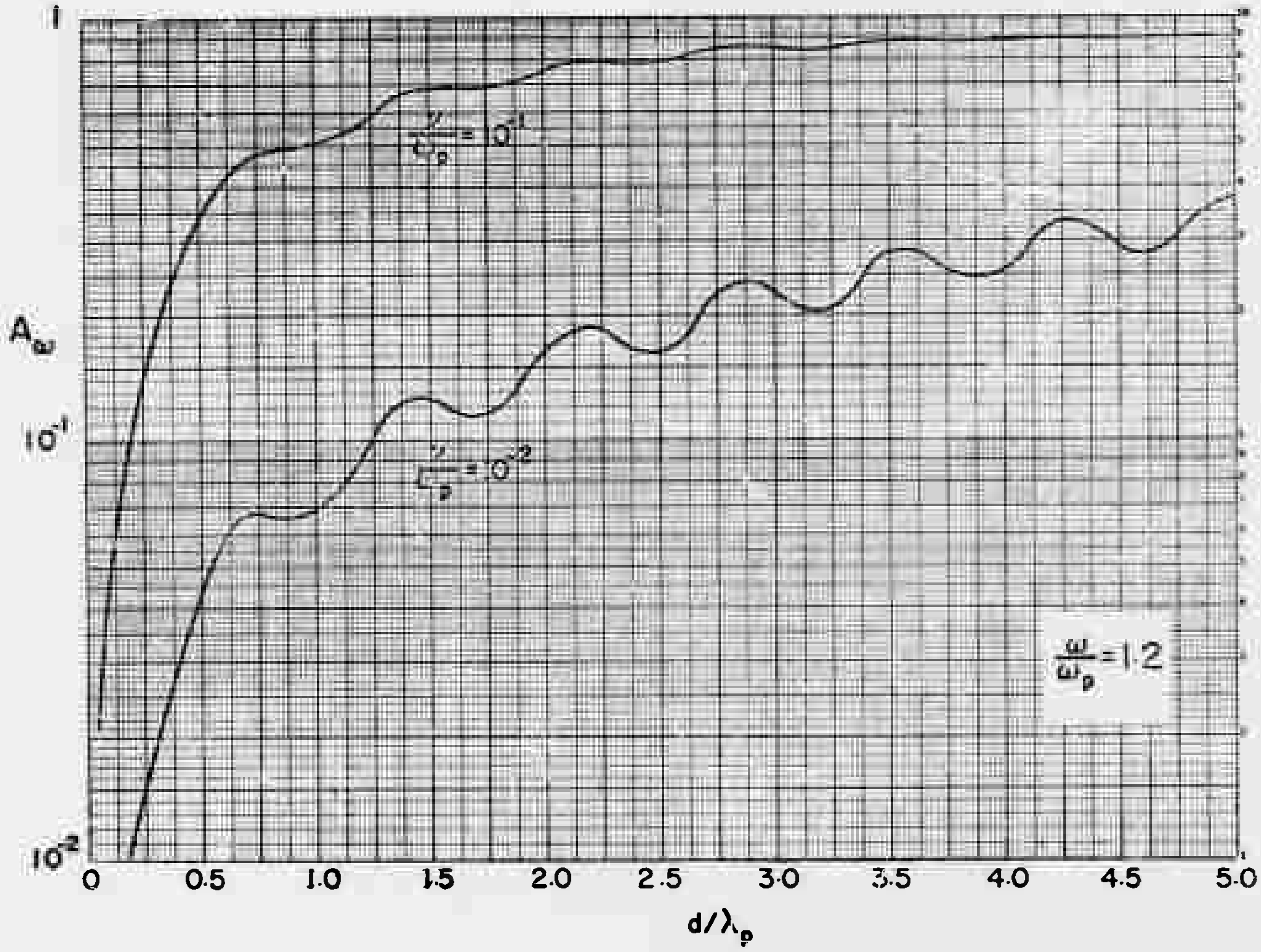

Fig. 3.11 Absorptivity versus slab thickness for ordinary wave for a frequency $\omega = 1.2\omega_p$.

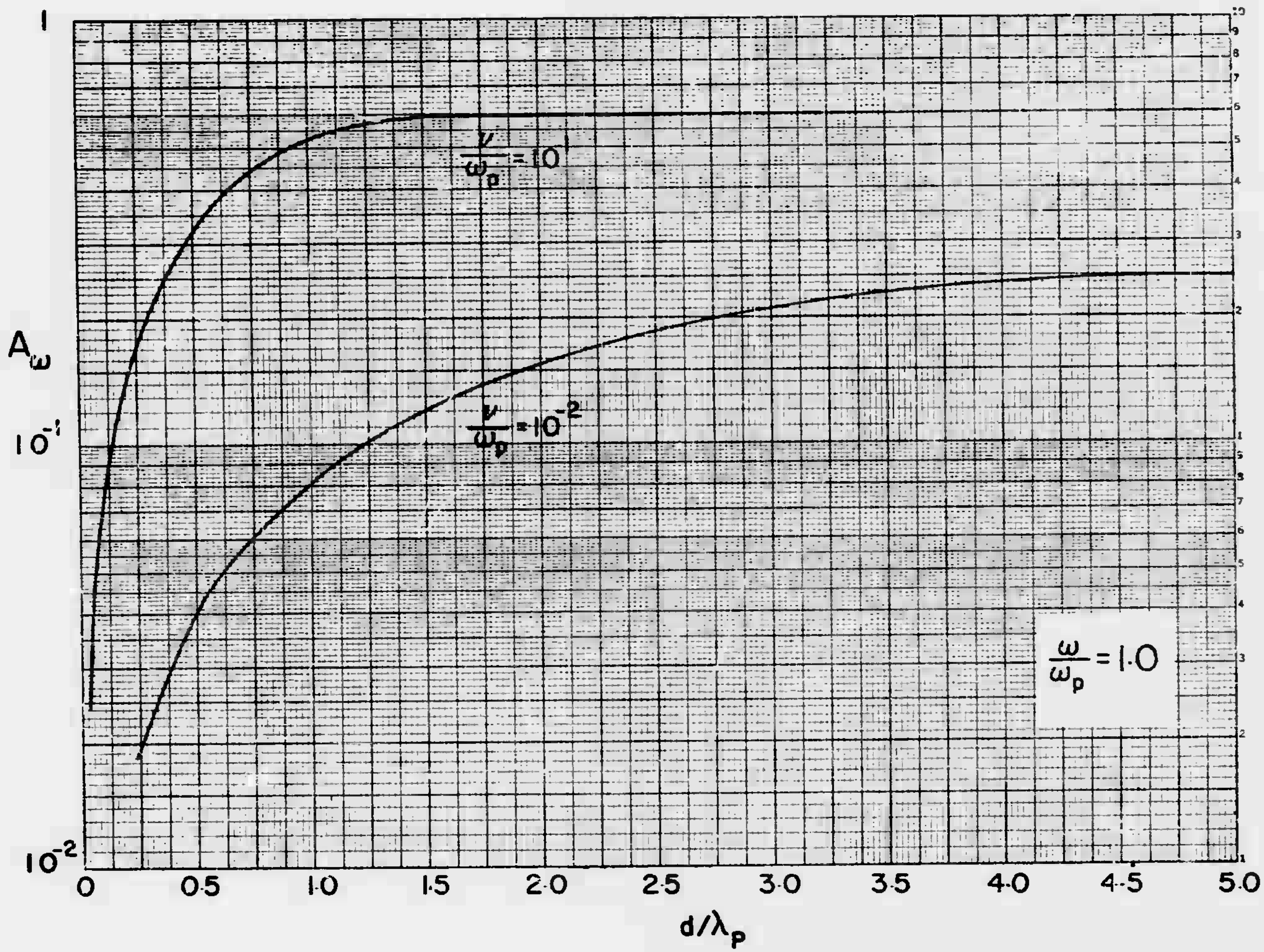

Fig. 3.12 Absorptivity versus slab thickness for ordinary wave for a frequency $\omega = 1.0\omega_p$.

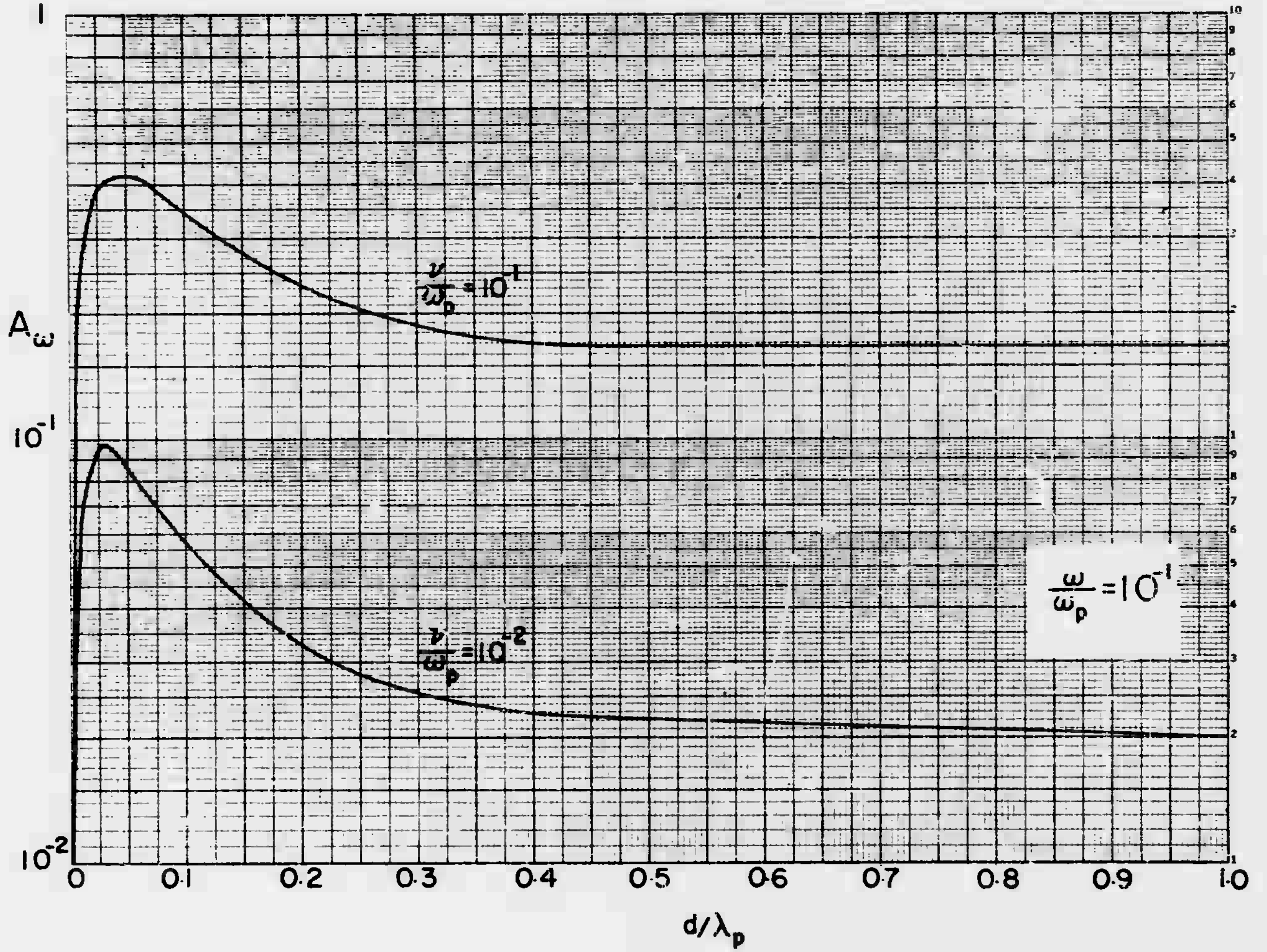

Fig. 3.13 Absorptivity versus slab thickness for ordinary wave for a frequency $\omega = .1\omega_p$.

IV. APPLICATIONS

It is possible that the passive radiation from the shock front of a re-entry vehiole will give information about this plasma sheath and subsequently about the vehiole itself. Whether the power received will be sufficient for a detailed analysis of the vehiole will depend on a large number of faotors, such as emissivity of the plasma, height of the vehicle, reoeiver sensitivity and the background noise. The latter is shown as a function of frequency in Fig. 4.1.[15] The densest part of the shock front is the stagnation region where electron densities may range[4] from about $10^{10}/cm^3$ to $10^{18}/cm^3$, corresponding to a plasma frequency (f_p) of 1 Kmc/s to 10^4 Kmc/s.

Kirchhoff's law (Eqn. 3.1) shows the power emitted by a body is proportional to the absorptivity and the square of the frequency. For the problem of calculating the passive microwave power emitted by a plasma sheath surrounding a re-entry vehicle, only the ordinary wave is considered, (neglecting the earth's magnetic field and any magnetic field the vehicle may carry). Fig. 3.10 shows that, in general, the radiation spectrum has a peak close to the plasma resonance, ω_p, and is independent of frequency higher in the spectrum. For maximum received power the radiometer should be tuned to the frequency range corresponding to maximum overall absorptivity, $A\omega$, of the plasma sheath. The power received at the radiometer is proportional to the power emitted, the aperture (gain) of the receiving antenna and the solid angle subtended by the plasma at the receiving antenna[16]. We can therefore write for the received power, P_r

$$P_r = P_e \Omega_s \sigma \tag{4.1}$$

where

$$P_e = \frac{KTA\omega}{2\pi\lambda^2} \quad \text{(Eqn. 3.1)} \tag{4.2}$$

is the emitted power, assuming the plasma to radiate isotropically.

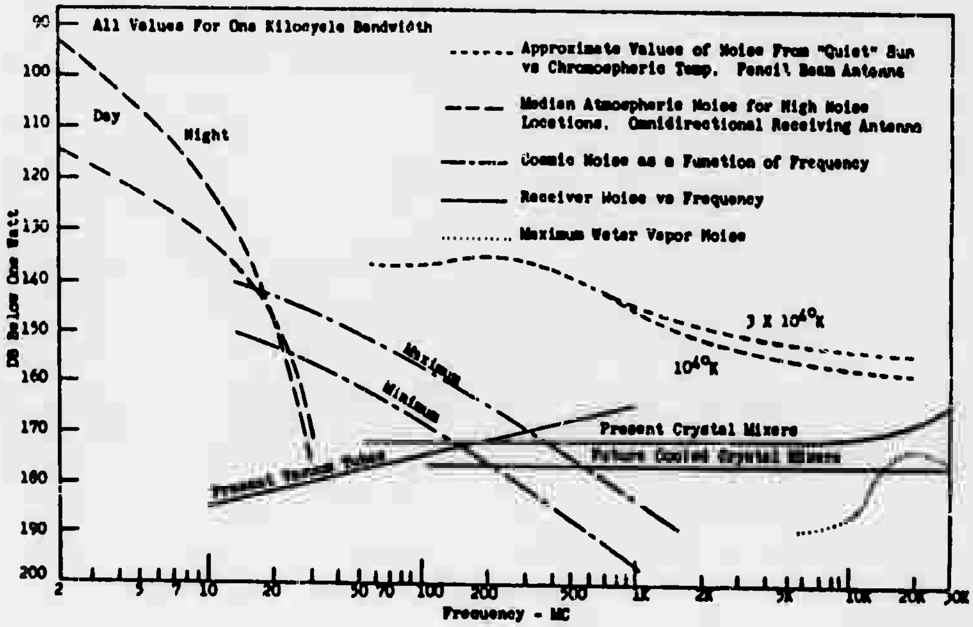

Fig. 4.1 Noise factors in communications.
(from reference 15).

Equation (4.1) becomes

$$P_r = \frac{KT}{2\pi\lambda^2} A_\omega \Omega_s \sigma$$

$\Omega_s = \frac{S_p}{R^2}$ is the solid angle subtended by the source at the antenna.

R is the distance from source to the antenna.

σ is the effective receiving antenna aperture.

S_p is the surface area of the plasma as seen by the radiometer.

Ω_r, the solid angle beamwidth $= \theta^2 \cong \lambda^2/D^2 = \lambda^2/\sigma$ of the receiving antenna.

where: θ = linear angular beamwidth.

D = antenna linear aperture.

Equation (4.1) becomes:

$$P_r = KTA_\omega \frac{\Omega_s}{\Omega_r}$$

Assuming $\Omega_s < \Omega_r$ i.e. that the plasma is wholly within the beamwidth of the receiving antenna, as will usually be the case in practice, the ratio Ω_s/Ω_r is independent of frequency and therefore the maximum of P_r and A_ω coincide. Due to the high electron densities possible in the plasma sheath it is possible that the overall maximum in A_ω will correspond to a frequency in the very high microwave spectrum, where atmospheric absorption is high. For example, atmospheric absorption of electromagnetic energy at 20 Kmc/s is about .3 db per mile and, in general, increases with frequency[17]. In such a case a compromise would be required.

The possibility of detecting cyclotron radiation from a shock plasma due to the earth's magnetic field is very unlikely. The cyclotron radiation due to electrons would be close to ω_b, which for the earth's magnetic field of approximately .4 gauss is about 1 Mc/s. The electron collision frequency will in general be much larger than this and so damp out the resonance. In addition, this frequency lies in the part of the spectrum where the atmospheric noise due to man-made and atmospheric sources is very high (see Fig. 4.1 from reference 15).

The addition of ion effects in the dielectric coefficient of a uniform anisotropic plasma reveals the existence of a very low frequency pass-band (when $\nu = 0$) at frequencies between zero and the ion cyclotron frequency. This opens up the possibility of using this pass-band for propagation through a plasma. A magnetic field might be carried in a hypersonic vehicle for this purpose. This part of the spectrum will, however, have low reflection and low attenuation only when the ion collision frequency is small compared to the ion cyclotron frequency. In general, the ion collision frequency is of the same order of magnitude as the electron collision frequency. Another effect of ions is the increase of absorptivity of the plasma in the region around Ω_b, as long as the effect is not damped out by the ion collision frequency.

The passive electromagnetic radiation from a plasma promises to be a useful diagnostic tool in determining plasma parameters. One of its main advantages would be that it does not perturb the plasma as do the more conventional techniques of metallic probes, electromagnetic waves and electron beams. The spectrum of radiation from a plasma may by comparison with curves of the type shown in section 3 yield information of the plasma frequency, electron collision frequency, the degree of inhomogeneity of the electron density and the applied magnetic field. Radiation has already found application as a diagnostic tool in fusion work where it represents a large energy loss[18]. The cyclotron radiation may also give information on the impurities due to sputtering from the metal walls enclosing a fusion plasma[19]. Calculations[20] show that if copper impurities make up 10^{-4} of the ions in the plasma the radiation loss due to bremsstrahlung increases by ten percent. The various impurities in the plasma should also give a peak in radiation at the ion cyclotron frequency Ω_b ($= eB/M$) of that particular species as long as it is present in sufficient quantities and Ω_b is much larger than the ion collision frequency.

The electromagnetic radiations from "space" have provided all our knowledge of universe. In recent years radio astronomy has added its contribution. Cyclotron (or synchrotron) radiation has provided information on the magnetic fields and plasmas of the sun, galaxies, nebula and other regions of "space". The polarization of cyclotron radiation has given an idea of the magnitude and orientation of magnetic lines of force as far away as 6,000 light years in the Crab Nebula[21]. It was found that the degree of polarization from this nebula is two thirds at radio frequencies[22].

The Faraday rotation principle may give rise to some useful microwave devices, analogous to ferrites, for propagation control. The non-reciprocal nature of Faraday rotation in ferrites has given rise to gyrators, attenuators and phase shifting devices. Gas discharge devices of this type could also be built. The Faraday rotation due to the earth's magnetic field on a re-entry shock plasma was investigated and found to be negligible. However, the Faraday rotation in the ionosphere, which, in general has a smaller electron concentration and has much larger thickness is shown to be appreciable as is well known from signal telemetry from high altitude rockets. This phenomenon has been used to obtain an integrated electron content for the ionosphere[23].

REFERENCES

1.	Bachynski, M.P. Shkarofsky, I.P. Johnston, T.W.	"Plasmas and the Electromagnetic Field" McGraw-Hill (in press).
2.	Astrom, E.	"On Waves in an Ionized Gas", Arkiv for Fysik 2, p. 443-460 (1951).
3.	Bachynski, M.P.	"Electromagnetic Wave Propagation", graduate course Electrical Engineering Dept., McGill University 1959-1960.
4.	Bachynski, M.P. Johnston, T.W. Shkarofsky, I.P.	"Electromagnetic Properties of High Temperaturo Air", Proc. IRE, Vol. 48, No. 3, p. 347, March, 1960.
5.	Erhleman, V.R. Peterson, A.M.	"Radar Astronomy", Scientific American, August, 1960.
6.	Etter, J.E. Goldstein, L.	"Guided Wave Propagation in Gyromagnetic Gaseous Discharge Plasmas", University of Illinois Electrical Engineering Research Laboratory Report No. 3 on Contract AF19(604)-524.
	Goldstein, L.	"Nonreciprocal Electromagnetic Wave Propagation in Ionized Media", IRE Trans MTT Vol. 6, p. 19, January 1958.
7.	Tellegen, B.D.H.	Nature, Vol. 131, p. 840, 1933.
8.	Anderson, J.M. Goldstein, L.	"Interaction of Microwaves in Gaseous Discharge Plasmas: Application to the Study of Fundamental Processes in Gases", E.E.R.L. University of Illinois Technical Report No. 7 on Contract No. AF19(604)-524.
9.	Ginsburg, V.L.	"Nonlinear Interaction of Radio Waves Propagating in a Plasma", J.E.T.P. 35(8) No. 6, p. 1100, June 1959.
10.	Rytov, S.M.	"Theory of Electrical Fluctuations and Thermal Radiations", Academy of Sciences Press, Moscow, (1953).
11.	Levin, M.L.	"The Electrodynamic Theory of Thermal Radiation", Dokl. Ahad. Nauk. (S.S.S.R.) 102, p. 53, 1955.

12. Bekefi, G.
Hirshfield, J.L.
Brown, S.C.
"Incoherent Microwave Radiation from Plasmas", Phys. Rev. 116, No. 5, p. 1051, December 1, 1959.

13. Dellis, A.N.
"The Measurement of Electron Temperatures by Microwave Methods", AERE GP/R 2265, Harwell, 1958.

14. Cloutier, G.G.
"Microwave Emission from Plasma - 1. Concept of Equilibrium Radiation and their Application", RCA Victor Research Report 1 to RCA Moorestown.

15. Perlman, S. et al
"Concerning Optimum Frequencies for Space Vehicles Communications", I.R.E. National Symposium on Extended Range and Space Communications, October, 1958.

16. Down-Range Anti-Ballistic Measurement Program (D.A.M.P.) "Microwave Radiometry Research Program. Description of Hypersonic Range Instrumentation". RCA Moorestown Report.

17. French, I.P.
Shkarofsky, I.P.
Bachynski, M.P.
"The Radio Spectrum Above 10 Kmc/s", RCA Victor Research Report 7-400,1, Montreal, July 1959.

18. Trubnikov, B.A.
Kudryavtsev, V.S.
"Plasma Radiation in a Magnetic Field", Second V.N. Conference on the Peaceful Uses of Atomic Energy. 1958 Vol. 32, Paper P/1330, Geneva, 1958.

19. Craston, J.L.
Hancox, R.
Robson, A.E.
Kaufman, S.
Miles, H.T.
Ware, A.A.
Wesson, J.A.
Second U.N. International Conference on Peaceful Uses of Atomic Energy Paper 15/P/34, Geneva, 1958.

20. Moore, W.J.
"The Ionic Bombardment of Solid Surfaces", American Scientist 48, No. 2, p. 109, June, 1960.

21. Oort, J.H.
"Radio Astronomy - A Window on the Universe" American Scientist 48, No. 2, p. 160, June, 1960.

22. Westfold, K.L.
"The Polarization of Synchrotron Radiation", Astrophysics Journal 130, No. 1, p. 241, July, 1959.

23. Prenatt, R.E.
"Ionospheric Structure Above Fort Churchill, Canada, from Faraday Rotation Measurements", A.R.S. Journal Vol. 30, No. 8, p. 763, August, 1960.